[redacted]

SHOP MATHEMATICS

SHOP MATHEMATICS

A TREATISE ON APPLIED MATHEMATICS DEALING WITH VARIOUS MACHINE-SHOP AND TOOL-ROOM PROBLEMS, AND CONTAINING NUMEROUS EXAMPLES ILLUSTRATING THEIR SOLUTION AND THE PRACTICAL APPLICATION OF USEFUL RULES AND FORMULAS

BY

ERIK OBERG
EDITOR OF MACHINERY
EDITOR OF MACHINERY'S HANDBOOK AND MACHINERY'S ENCYCLOPEDIA
AUTHOR OF "HANDBOOK OF SMALL TOOLS," ETC.

AND

FRANKLIN D. JONES
ASSOCIATE EDITOR OF MACHINERY
AUTHOR OF "TURNING AND BORING," "PLANING AND MILLING," "MECHANISMS AND MECHANICAL MOVEMENTS," "THREAD-CUTTING METHODS," ETC.

FIRST EDITION
FIRST PRINTING

NEW YORK
THE INDUSTRIAL PRESS
LONDON: THE MACHINERY PUBLISHING CO., LTD.
1920

COMPOSITION AND ELECTROTYPING BY THE PLIMPTON PRESS, NORWOOD, MASS., U.S.A.

PREFACE

THIS book has been published primarily for machinists and toolmakers, and for students interested in what is commonly known as "shop mathematics." The object of the book is to explain and illustrate, by numerous examples, the different kinds of problems which are commonly encountered in the construction of machinery and tools. The selection of the subjects treated is based on a careful study of the problems which are of the greatest practical value, and often the most perplexing to the average shop man. The examples have not been confined entirely to shop problems, in order that a somewhat broader field might be covered and important principles explained.

Many of the more elementary subjects, such as addition, multiplication, division, etc., have been omitted, as practically all skilled men and most apprentices possess this rudimentary knowledge. But, in view of the fact that calculating is not a regular part of the shop man's work and he is liable to become "rusty" even in some of the simpler branches of arithmetic, a few elementary subjects which often prove "stumbling blocks" have been included, such, for example, as the multiplication and division of common and decimal fractions, proportion, involution and evolution, and percentage.

While drawings should be so complete that calculations in the shop or tool-room are, as a general rule, unnecessary, in small shops, and sometimes in connection with special work, the machinist or toolmaker often finds it desirable to solve his own problems; and even though mathematics is not applied directly to the work of the shop, a knowledge of this subject will usually greatly assist the man who desires to advance. In fact, many excellent designers as well as foremen and superintendents are shop graduates who studied mathematics. This book, therefore, is intended not only to assist in the solution of the problems liable to arise in everyday shop practice, but to lay the foundation for a higher position in manufacturing and engineering practice. The men whose training has been confined largely to the machine shop and tool-room often find it difficult to apply mathematical theories to shop problems. Because of this fact, most of the examples in this book are taken directly from the shop, and they have been largely selected from problems which shop men have repeatedly submitted for solution. Another feature of the book which is considered important is that numerous examples are included so that the method of actually applying a rule or formula will be entirely clear.

THE AUTHORS.

New York, *March*, 1920.

CONTENTS

SHOP MATHEMATICS

CHAPTER I

VALUE OF MATHEMATICS TO MACHINISTS AND TOOLMAKERS

THE design of machines depends very largely upon mathematical calculations, all engineering practice being based upon physics and mathematics. While the machinist and toolmaker do not require the mathematical knowledge of the designer or mechanical engineer, all mechanical work is so closely related to mathematics that even the man in the shop often finds a practical working knowledge of this subject useful and sometimes indispensable. But even though all problems are solved outside of the shop, a knowledge of the more important and useful branches of mathematics will prove of great value to shop men who desire to fit themselves for a higher position either as a foreman or superintendent or as a draftsman and designer.

There are numerous opportunities in the mechanical field within the grasp of men in the shop, although many may be sceptical regarding the accuracy of such a statement. That it is true, however, is indicated by the fact that almost invariably the machinist who devotes his spare time to study is advanced to positions of responsibility. This does not mean that success always follows study, nor that one can completely fit himself for a responsible position by reading and study. Books give much that is essential, but cannot supply all. A man's personality, his character, his temperament, his judgment — all of these are factors which determine the degree of his success; but, without knowledge, these in themselves are insufficient. The foreman, draftsman, or superintendent, each must possess the personal qualifications which fit him for his position. Nevertheless, the machinist whose ambition impels him to the reading and study of books and publications that explain the principles and practice of mechanics, usually is promoted; and this important fact deserves wider recognition than it receives among most shop men.

Importance of Study of Shop Mathematics. — Without the aid of the processes of arithmetic, even the simplest mechanical work could hardly be done. In the design of machinery, and still more in the design of great engineering structures, calculations of a more or less advanced nature become absolutely necessary. Any mechanic with a limited education who contemplates the study of mathematics should make certain that he has fully mastered arithmetic. Just as mathematics is the basic science underlying engineering, so arithmetic is the basis of all mathematics. Without a thorough understanding of every process in arithmetic, other mathematical studies become difficult, if not impossible.

Many shop men refrain from using handbooks and other mechanical books containing formulas, because they believe that an understanding of algebra is necessary in order to make use of such formulas as are given in handbooks. This idea is erroneous, as will be apparent after reading carefully that part of Chapter III dealing with the use of formulas. With few exceptions, the formulas given in handbooks intended for machine shops can be used by anyone who thoroughly understands arithmetic. In mathematics, a number of abbreviations, signs, and symbols are also used; and it is of considerable value to the man who reads mechanical literature and occasionally uses formulas to memorize the commonly used signs and abbreviations. This will facilitate his progress and make it easier for him to grasp the meaning of a formula which otherwise would be obscure.

Closely allied to the use of formulas is the use of diagrams. A formula records a mathematical statement by means of symbols or letters, while a diagram records a similar statement graphically by means of lines. Many mechanics regard a diagram as something difficult to understand, but this is not the case, as anyone can easily find by studying a few diagrams such as are presented in Chapter XVII of this book.

The student who wishes to go further into the study of elementary mathematics should begin with a simple course in the solution of triangles and elementary geometry. If he wishes to proceed still further, he should take up logarithms and the solution of equations, and in connection with the latter subject he would acquire the rudiments of algebra.

Mathematics in the Tool-room. — The value of clearly understanding the underlying principles of simple mathematics and trigonometry is not always evident to the toolmaker. He often relies on makeshift or graphical methods to overcome a difficulty when any question involving a mathematical calculation comes up. The cut-and-try process seems to many toolmakers the only practical way of solving any problem that varies slightly from the ordinary run of tool work. He will spend hours arranging buttons to find the position of certain holes which could be quickly determined by a simple mathematical calculation and direct measurement. This point is illustrated by the following incident:

A toolmaker was given the job of laying out a multiple punch and die for blanking and cupping six blanks located diagonally in the strip, at one stroke of the press. The problem was to find the width of strip stock required to punch out six blanks economically. From experience this toolmaker knew that to obtain the greatest number of blanks from a given area the holes or circles should be laid out diagonally, and he set out to find their positions. Not realizing that they could be determined mathematically, he tried to find them by the button method, setting five buttons in a group having diameters equal to the blank plus the web. This took over a day, as the position of each hole had to be determined progressively; but a few minutes would have sufficed had he understood the solution of right-angled triangles.

Where the work is laid out in the designing department and all essential dimensions given on drawings, it is not so necessary that the toolmaker have a knowledge of mathematics as it is when he has to lay out the work for himself as he goes along. Many toolmakers point to this fact, stating that a knowledge of mathematics is useless to them. But they overlook the fact that a toolmaker who has a knowledge of mathematics is unlikely to remain a toolmaker always, because his knowledge fits him for higher positions which he could not satisfactorily fill without it. Practical experience is absolutely necessary, but when coupled with a good technical education it is far more valuable to the possessor.

CHAPTER II

ARITHMETIC COMMONLY USED IN SHOP PROBLEMS

THIS chapter deals with certain branches of arithmetic which should be thoroughly understood since they are often required in solving problems which are commonly encountered in mechanical work. The most elementary subjects in arithmetic, such as plain addition, subtraction, division, and multiplication, have been omitted, the assumption being that practically none of the users of this book needs such instruction. The multiplication and division of common and decimal fractions have been explained and also cancellation, proportion, percentage, square root, cube root, and a few other subjects which are sometimes perplexing especially to those whose work has largely been confined to the machine shop or tool-room. All of the subjects covered in this chapter should be clearly understood before attempting to study the following chapters or to solve the problems which they contain.

Multiplication and Division of Fractions. — The rules for the multiplication and division of common fractions will be given for the benefit of the few who may not be familiar with this part of elementary arithmetic, as many of the problems in this book require the use of fractions.

Two fractions are multiplied by multiplying numerator by numerator and denominator by denominator (*numerator* being the *upper*, and *denominator* the *lower* quantity in a fraction). For instance, let it be required to multiply $\frac{1}{4}$ by $\frac{3}{8}$. We have then,

$$\frac{1}{4} \times \frac{3}{8} = \frac{1 \times 3}{4 \times 8} = \frac{3}{32}.$$

If the numbers to be multiplied contain whole numbers, these are first converted into fractions. Let it be required to multiply $1\frac{1}{4}$ by $3\frac{1}{4}$. We have then,

$$1\frac{1}{4} \times 3\frac{1}{4} = \frac{5}{4} \times \frac{13}{4} = \frac{65}{16} = 4\frac{1}{16}.$$

Division is simply the reverse of multiplication. The number which is to be divided is called the *dividend*, and the number by which we divide is called the *divisor*. If one number is to be divided by another, simply invert the divisor, and *proceed as in multiplication*. To invert the divisor means that we place the denominator as numerator, and the numerator as denominator; for instance, $\frac{3}{8}$, inverted, is $\frac{8}{3}$. Suppose that we wish to divide $\frac{3}{4}$ by $\frac{7}{16}$. We have then,

$$\frac{3}{4} \div \frac{7}{16} = \frac{3}{4} \times \frac{16}{7} = \frac{48}{28} = 1\frac{20}{28} = 1\frac{5}{7}.$$

If the number to be divided contains a whole number besides a fraction, we first convert this into a fraction, and then proceed as before. Suppose that we wish to divide $2\frac{1}{4}$ by $3\frac{3}{4}$. We have then,

$$2\frac{1}{4} \div 3\frac{3}{4} = \frac{9}{4} \div \frac{15}{4} = \frac{9}{4} \times \frac{4}{15} = \frac{36}{60} = \frac{3}{5}.$$

Cancellation. — Cancellation is the process of taking out equal factors in both the numerators and denominators of fractions to be multiplied, and is used for simplifying the work of multiplication of fractions. If the fractions $\frac{5}{6} \times \frac{18}{20}$ are to be multiplied, the work can be done more easily and quickly by first cancelling factors in the numerators and denominators as far as possible. If a dividend and divisor are both divided by the same number, this does not change the quotient. In the preceding example, the numerator 5 and the denominator 20, or $\frac{5}{20}$, are equal to $\frac{1}{4}$, which is obtained when 5 and 20 are each divided by 5. In the same way, the denominator 6 and the numerator 18 can be reduced to 1 and 3, respectively. The common method of procedure is to draw a line through 5 and 20 and place 1 above the 5 and 4 below the 20. The figure 6 is also crossed out and replaced by 1 and 18 by 3. Thus:

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} \times \frac{\overset{3}{\cancel{18}}}{\underset{4}{\cancel{20}}} = \frac{3}{4}.$$

The final result is obtained in this way without multiplying 5 by 18 and 6 by 20. Another example of cancellation follows:

Multiply $\frac{4}{15} \times \frac{3}{12} \times \frac{6}{11} \times \frac{33}{40}.$

This can be carried out as shown below:

$$\frac{\overset{1}{\cancel{4}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{12}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{\underset{1}{\cancel{11}}} \times \frac{\overset{3}{\cancel{33}}}{\underset{20}{\cancel{40}}} = \frac{3}{100}.$$

Multiplication of Decimal Fractions. — When multiplying decimal fractions, the multiplier is placed under the multiplicand, in the same way as in multiplication of whole numbers. While carrying out the multiplication, no attention is paid to the decimal point; the numbers are simply placed in such a manner that the right-hand figure in the multiplicand comes directly over the right-hand figure in the multiplier. It makes no difference whether the decimal points should happen to come under each other or not. If 126.5623 is to be multiplied by 4.67, write the numbers thus:

```
 126.5623
     4.67
---------
  8859361
 7593738
5062492
---------
591.045941
```

The multiplication is carried out exactly as when whole numbers are to be multiplied. The number of decimal places in the product equals the sum of the number in the multiplicand and the multiplier. If there are four decimals places in the multiplicand, and two decimal places in the multiplier, as in this example, then there should be six decimal places in the product, and we place the decimal point in the product so that there are six figures to the right of the decimal point.

Rule: Place the multiplier under the multiplicand, disregarding the decimal point. Multiply as in whole numbers, and in the product, point off as many decimal places as there are decimal places in both the multiplier and multiplicand. If there are not enough figures in the product to point off the required number of decimal places, prefix ciphers, put in the decimal point, and place a cipher to the left of the decimal point to indicate that there is no whole number.

Division of Decimal Fractions. — When dividing decimal fractions, the dividend, divisor, and quotient are placed in the same manner as in the division of whole numbers. If there are not as many decimal places in the dividend as in the divisor, add ciphers to the one having the smaller number of decimal places, until there is an equal number, and then divide as whole numbers, disregarding the decimal point.

Example. — Divide 3.25 by 0.0625.

dividend 3.2500	0.0625 divisor
3125	52 quotient
1250	
1250	

In the example above, there are two decimal places in 3.25 and four in 0.0625. Therefore add ciphers to 3.25 until there are four decimals in that number, thus: 3.2500. Now divide as when dividing whole numbers, and pay no attention to the decimal point. If there should be a remainder, the division would be continued as in the example below:

Example. — Divide 23.1875 by 0.25.

23.1875	0.2500
22500	92.75
6875	
5000	
18750	
17500	
12500	
12500	

If there is a remainder when the last figure has been brought down from the dividend, place a decimal point after the figures already obtained in the quotient, annex a 0 to the remainder left from the last subtraction, and continue to divide as before. To each remainder obtained annex a 0. This 0 takes the place of the figure brought down from the dividend. The figures obtained in the quotient after the decimal point has been placed are decimals.

Dividing an Odd Number and a Fraction. — It is sometimes necessary to divide inches and fractional parts of an inch by 2, as when laying off center lines, spacing, etc. The following method will be found very convenient:

Rule: To divide an odd number and a fraction by 2, add together the numerator and denominator of the fraction, placing the sum in position for the numerator of the fraction of the quotient. For the denominator of this fraction, multiply by 2 the denominator of the fraction which is being divided. Deduct 1 from the whole number of the expression that is being divided, and divide the remainder by 2.

Example. — Divide $31\frac{31}{32}$ by 2. The numerator of the fraction added to the denominator equals $31 + 32 = 63$, which is the numerator of the fraction of the quotient. The denominator equals $32 \times 2 = 64$. The whole number in the quotient equals $(31 - 1) \div 2 = 15$; hence, $\frac{31\frac{31}{32}}{2} = 15\frac{63}{64}$

Proving Multiplication. — The method of proving multiplication ordinarily used is that of casting out 9's. This is effected by dividing the multiplicand and multiplier by 9 and noting the remainders, which are then multiplied together and divided by 9; if the remainder thus obtained is the same as the remainder obtained by dividing the product by 9, the work is probably correct; but, if it is not, the work is wrong. Thus, $7854 \times 2905 = 22{,}815{,}870$. Here $7854 \div 9$ gives a remainder of 6; $2905 \div 9$ gives a remainder of 7; $6 \times 7 = 42$, and $42 \div 9$ gives a remainder of 6; $22{,}815{,}870 \div 9$ gives a remainder of 6, and the work is probably correct. This test is not always certain, since the remainder, when dividing by 9, may always be obtained by adding the digits, then adding the digits of the sum, etc., until a single figure is obtained; hence, if one or more mistakes are made whereby the sum of the digits (reduced to a single figure) is unchanged, the test fails. Thus, the remainders obtained by dividing the foregoing numbers by 9 are, respectively, $7 + 8 + 5 + 4 = 24$, and $2 + 4 = 6$; $2 + 9 + 5 = 16$, and $1 + 6 = 7$; $4 + 2 = 6$; and $2 + 2 + 8 + 1 + 5 + 8 + 7 = 33$, and $3 + 3 = 6$. If the product obtained had been 22,815,780, 23,805,870, 22,814,970, or any one of numerous other combinations in which the sum of the digits when reduced to one figure is 6, it is evident that the test would fail.

A much better test, and one that is practically certain, is to divide by 7. Thus, $7854 \div 7$ gives a remainder of 0; here without proceeding further it is known at once that the product when divided by 7 must give a remainder of 0, since one of the factors being a multiple of 7, the product is a multiple of 7. Dividing 22,815,870 by 7, the remainder is 0, showing that the work is correct. Consider the product $7853 \times 2904 = 22{,}805{,}112$. Here $7853 \div 7$ gives a remainder of 6; $2904 \div 7$ gives a remainder of 6; $6 \times 6 = 36$, $36 \div 7$ gives a remainder of 1. Since $22{,}805{,}112 \div 7$ gives a remainder of 1, the work is correct. The reader should apply the 7 test to the preceding numbers that were apparently correct by the 9 test, but were wrong in reality.

Another method of proving multiplication is illustrated by the following example:

Example. — Multiply 84,689 by $5214 = 441{,}568{,}446$. Add all the digits of the multiplicand till one digit is obtained, thus: $8 + 4 + 6 + 8 + 9 = 35$, and $3 + 5 = 8$. Do likewise with the multiplier, thus: $5 + 2 + 1 + 4 = 12$, and $1 + 2 = 3$. Multiply the two results and add the digits till one digit is obtained: $8 \times 3 = 24$, and $2 + 4 = 6$. Lastly, add the digits of the product till one digit is obtained; thus: $4 + 4 + 1 + 5 + 6 + 8 + 4 + 4 + 6 = 42$, and $4 + 2 = 6$. The result should agree with the result obtained by adding the digits of the preceding multiplication. In this case, the number is 6 in both cases, indicating that the product is correct.

Proving Division. — In order to prove division, the digits of the divisor, dividend, quotient (and remainder, if any) are added separately until one digit is obtained in each case, as previously described for proving multiplication. The product of the digits representing the divisor and the quotient is next multiplied and the result reduced to one figure. Now add the remainder, if any, and reduce the sum to one figure. If the division is correct, this final digit will be the same as the one representing the dividend.

Example. — $441{,}568{,}446 \div 84{,}689 = 5214$.

Proof: Add all the digits of the divisor; thus, $8 + 4 + 6 + 8 + 9 = 35$, and $3 + 5 = 8$. The sum of the digits of the dividend equals $4 + 4 + 1 + 5 + 6 + 8 + 4 + 4 + 6 = 42$, and $4 + 2 = 6$. The sum of the digits of the quotient equals $5 + 2 + 1 + 4 = 12$, and $1 + 2 = 3$. The final digit of the divisor multiplied by the final digit of the quotient equals $8 \times 3 = 24$, and $2 + 4 = 6$, which is the same as the figure representing the final digit of the dividend; therefore, the division is correct.

Example. — $93{,}279 \div 464 = 201\frac{15}{464}$.

Proof: Add all the digits of the divisor; thus, $4 + 6 + 4 = 14$, and $1 + 4 = 5$. The sum of the digits of the dividend equals $9 + 3 + 2 + 7 + 9 = 30$, and $3 + 0 = 3$. The sum of the digits of the quotient equals $2 + 0 + 1 = 3$. The sum of the digits of the remainder equals $1 + 5 = 6$. The final digit of the divisor multiplied by the final digit of the quotient equals $5 \times 3 = 15$, and $1 + 5 = 6$. This digit plus the final digit of the remainder equals $6 + 6 = 12$, and $1 + 2 = 3$, which is the same as the final digit representing the dividend; hence, the division is correct.

Proportion. — Two quantities are said to be in *direct proportion* when they bear such a relation to each other that as one is increased the other becomes greater, or, as one is decreased, the other becomes less at the same rate. The relation between the circumference of round bar stock and its diameter is an example of direct proportion. If the diameter increases, the circumference will increase, and if the diameter is made less, the circumference will be less.

If the relation between two quantities is such that as the one increases the other becomes smaller, and as the one decreases the other becomes greater in the same rate, they are in *inverse proportion*. The time required to build a machine shop is inversely proportional to the number of men employed, and the greater the number, the shorter the time.

Two quantities are said to be in *compound proportion* when the relation between them is such that the increase or decrease of one affects the other by a combination of two or more direct or inverse proportions. If one man can mill 50 steel castings in a day of 10 hours, then 5 men can mill 225 similar castings in 9 hours. The number of castings milled by one man in 10 hours is in compound proportion to the number milled by 5 men in 9 hours, because the proportion is a combination of the proportion between the number at work and the proportion of the time they are working.

In calculations, a proportion is usually written as below:

$$5 : 6 :: 10 : 12$$

which is read: five is to six as ten is to twelve.

In every proportion of four terms the product of the two extreme or outside terms equals the product of the two mean or intermediate terms; thus, in the proportion $5 : 6 :: 10 : 12$, the product 5×12 equals the product 6×10.

In a proportion, the sign (:) can be substituted by the division sign (÷), and the sign (::) by the equal sign (=), so that the proportion above may be written $5 \div 6 = 10 \div 12$ or $\frac{5}{6} = \frac{10}{12}$. The fraction on either side of the equal sign reduced to its lowest terms is called the *ratio* of the proportion.

In the example above, the fraction $\frac{5}{6}$ is already reduced to its lowest terms, so that $\frac{5}{6}$ is the ratio.

Examples of Direct Proportion. — If a gang of men work 14 days in assembling 6 milling machines, how long would it require to assemble 18 milling machines?

The time required to assemble 18 milling machines is directly proportional to the time required for 6 milling machines. If it takes 14 days to assemble 6 milling machines, it takes $\frac{14}{6} = 2\frac{1}{3}$ days for one milling machine, and $18 \times 2\frac{1}{3} = 42$ days to assemble 18 milling machines.

This problem could also be solved by writing out the proportion as below, the number of days to be found being represented by x:

$$6 : 14 :: 18 : x$$

(milling machines : days : : milling machines : days)

which is read, 6 is to 14 as 18 is to x. The problem now is to find the value of x.

As the product of the extreme terms in a proportion equals the product of the intermediate terms, therefore,

$$6 \times x = 14 \times 18.$$

If $6 \times x$, or $6x$, as it is commonly written, equals 14×18, then one $x = \frac{14 \times 18}{6} = 42$ days.

Example. — Thirty-four linear feet of bar stock are required for the blanks for 100 clamping bolts. How many feet of stock would be required for 912 bolts?

x = total length of stock required for 912 bolts.

$34 : 100 :: x : 912.$

$34 \times 912 = 100\,x.$

$\frac{34 \times 912}{100} = x.$ $x = 310$ feet, almost exactly.

It should be noted in the examples above that the position of x in the proportion depends upon the requirements of the problem. In every direct proportion it is necessary to have the corresponding quantities occupy the same relative place on each side of the proportion or equal sign. In the example just given, we have, for instance,

$$34 : 100 :: x : 912$$

feet : pieces : : feet: pieces

Examples of Inverse Proportion. — A shop equipped with 16 automatic screw machines produces a certain number of duplicate parts in a day of 10 hours. How many automatic screw machines would be required for the same production if the machines were only operated 8 hours each day?

In this example, the hours per day are in inverse proportion to the number of screw machines used; the shorter the time, the more screw machines are required. The example can be solved by the method explained previously; x is the number of automatic screw machines working 8 hours. The inverse proportion is written

$$16 : x :: 8 : 10$$

$$\left\{\begin{matrix}\text{machines operating}\\ \text{10 hours}\end{matrix}\right\} : \left\{\begin{matrix}\text{machines required}\\ \text{for 8-hour day}\end{matrix}\right\} :: \left\{\begin{matrix}\text{hours}\\ \text{per day}\end{matrix}\right\} : \left\{\begin{matrix}\text{hours}\\ \text{per day}\end{matrix}\right\}$$

Note that in an inverse proportion the corresponding quantities occupy inverse or opposite places in the proportion. Carrying out the calculation, we have

$$16 \times 10 = 8\,x;\ x = \frac{16 \times 10}{8} = 20.$$

Therefore, if the operating time is reduced from 10 hours to 8 hours, the number of machines must be increased from 16 to 20, if the same daily output is to be maintained.

Compound Proportion. — The kind of problems occurring in compound proportion is illustrated by the following example:

Example. — If a man capable of drilling 40 forgings in a day of 10 hours is paid 36 cents per hour, how much ought a man be paid who drills 48 forgings in an 8-hour day, if compensated in the same proportion?

When solving problems involving compound proportion, the following method of analysis tends to simplify the solution. Make up a table with four columns headed, "First Cause," "First Effect," "Second Cause," "Second Effect," and place under each the respective factors given in the problem. In the example above, the table would be arranged as below:

First Cause	*First Effect*	*Second Cause*	*Second Effect*
1 man	40 forgings	1 man	48 forgings
10 hours		8 hours	
36 cents		x cents	

Consider as *causes* the number of men working, the length of time they work, and their capacity for work; the pay received or the amount of product turned out in a unit of time indicates the capacity for work. The effect is the total product given either in numbers, or by the dimensions of the work completed. The unknown quantity is called x.

When the different numbers or quantitites have been arranged in columns as described, take all the quantities in the first and fourth columns and place them as the numerator of a fraction with multiplication signs between them, and all the quantities in the second and third columns and place them as the denominator of a fraction with multiplication signs between them. Make this fraction equal to 1. Then cancel and reduce the fraction to its simplest form as below

$$\frac{1 \times 10 \times 36 \times 48}{40 \times 1 \times 8 \times x} = 1.$$

$$\frac{54}{x} = 1, \text{ or } x = 54 \text{ cents.}$$

Example. — Fifteen turret lathes of a certain make turn out a total of 270 pieces per hour. It is planned to double the total product per day by installing machines of more modern type, each capable of producing 25 pieces per hour. At the same time, the working hours per day are to be reduced from 10 to 9. How many machines of the new type will be required to double the daily output?

It will be noted in this problem that the capacity of the new machines is given in production of each machine per hour, while the capacity of the old machines is given as the production of the total number of machines per hour. It is necessary that the capacity of the old machines be given in the same form as the capacity of the new machines. As 15 machines produce 270 pieces per hour, each machine produces $270 \div 15 = 18$ pieces per hour. Note that the capacities of the respective machines, 18 and 25 pieces per hour, are "causes" of their total production.

Another of the given conditions is that the total daily output should be doubled. As 270 pieces are now produced per hour, and the working day is 10 hours, the total daily production is $270 \times 10 = 2700$. Double this number, or 5400 pieces, is the required output per day of the new equipment. Having obtained these figures, we can now tabulate the conditions.

First Cause	*First Effect*	*Second Cause*	*Second Effect*
15 machines	2700 pieces	x machines	5400 pieces
10 hours		9 hours	
18 pieces		25 pieces	

Following the same method as shown above, we have:

$$\frac{15 \times 10 \times 18 \times 5400}{x \times 9 \times 25 \times 2700} = 1;$$

$$\frac{24}{x} = 1, \qquad \text{or} \qquad x = 24 \text{ machines.}$$

Proportion Involving Powers and Roots. — In some problems, the quantities or value may not vary directly in proportion to some other value, but according to the power or the root of that value. For example, the area of a circle varies as the square of the diameter and the volume and weight of a sphere varies as the cube of the diameter.

The procedure, when either powers or roots of numbers occur in proportion, is illustrated by the following example:

Example. — If a solid cast-iron ball 20 inches in diameter weighs 1090 pounds, what is the weight of a ball 16 inches in diameter made of the same material?

As the weight of a sphere varies as the cube of the diameter, the proportion must be based on the cubes of the two diameters given and not directly on the diameters. Thus:

$$20^3 : 16^3 :: 1090 : x.$$

Expressed in words, the cube of 20 is to the cube of 16 as 1090 is to x. Instead of finding the cube of 20 and of 16, these terms of the ratio may be reduced without changing the value of the ratio, by dividing both terms by the same number. For example, the ratio of 20 cubed to 16 cubed equals the ratio of 5 cubed to 4 cubed. Therefore,

$$5^3 : 4^3 :: 1090 : x, \quad \text{or} \quad 125 : 64 :: 1090 : x.$$

Hence, $$x = \frac{64 \times 1090}{125} = 558 \text{ pounds.}$$

Example. — If a solid cast-iron ball weighs 124 pounds and is 10 inches in diameter, what would be the diameter of a similar ball weighing 100 pounds?

In this problem, the cubes of the diameters must again be considered. Since one diameter is not known and is represented by x, the proportion is

$$124 : 100 :: 10^3 : x^3.$$

Therefore, $$x^3 = \frac{100 \times 10^3}{124} = \frac{100 \times 1000}{124} = 806.4, \text{ nearly.}$$

Since $x^3 = 806.4$, the value of $x = \sqrt[3]{806.4} = 9.3$ inches.

Figuring Percentage. — The term "percentage" is applied to numerical operations based upon one hundred as a unit of computation. Ten per cent of a number is equal to ten-hundredths of that number. Thus, ten per cent of one hundred equals $\frac{10}{100} \times 100 = 10$. The per cent of a number is usually expressed as a decimal instead of as a common fraction. For instance, four per cent, or 4% (the sign of per cent is %), is ordinarily written as 0.04 in preference to $\frac{4}{100}$. The general rule for percentage calculations will be given and then the practical application of percentage will be illustrated by some typical problems.

Rule: To find the per cent of gain or loss, divide the amount of gain or loss by the *original* number of which the percentage is wanted, and multiply the quotient by 100.

Example. — A turret lathe operator produces 320 parts in a week, but 40 parts do not pass inspection. What is the percentage of rejected parts?

The original number of parts is 320; therefore, the percentage of rejected parts is obtained by dividing the loss, or 40, by 320 and multiplying by 100. Thus:

$$\text{Percentage of loss equals } \frac{40}{320} \times 100 = 12.5\%.$$

Example. — If 60 parts are finished on a planer in a day and the production is increased to 90 parts by using a horizontal milling machine instead of the planer, what is the gain expressed in per cent?

In this example, the amount gained equals $90 - 60 = 30$, and the original number is 60; therefore, the gain in per cent is obtained by dividing 30 by 60 and multiplying by 100. Thus:

$$\text{Percentage of gain equals } \frac{30}{60} \times 100 = 50\%.$$

In solving problems of this kind, it is important to use as the divisor the *original number*, or the number of which the percentage is wanted. In the preceding example, the percentage of gain over the original output of 60 was wanted, and not the percentage in relation to the new rate or production, or 90, and, therefore, the number 30, representing the gain, is divided by 60.

Powers of Numbers. — The product obtained by multiplying a number by itself one or more times is known as the "power" of that number. The second power of 5 equals 5 multiplied by itself, or $5 \times 5 = 25$, and the third power of 5 equals $5 \times 5 \times 5 = 125$. The second power is commonly called the *square* of the number, and the third power, the *cube*.

The square of 2 is $2 \times 2 = 4$, and the square of 10 is $10 \times 10 = 100$; similarly the square of 177 is $177 \times 177 = 31,329$. Instead of writing 2×2 for the square of 2, it is often written 2^2, which is read "two square," and means that 2 is multiplied by 2. In the same way, 128^2 means 128×128. The small figure (^{2}) in these expressions is called *exponent*.

The cube of a number is the product obtained if the number itself is repeated as a factor three times. The cube of 2 is $2 \times 2 \times 2 = 8$, and the cube of 12 is $12 \times 12 \times 12 = 1728$. Instead of writing $2 \times 2 \times 2$ for the cube of 2, it is often written 2^3, which is read "two cube." In the same way 128^3 means $128 \times 128 \times 128$. The small figure (^{3}) in this case is the *exponent*. An expression of the form 18^3 may also be read the "third power of 18."

The exponent in the expression 6^4 indicates the fourth power of 6, the exponent showing in each case how many times the number to which the exponent is affixed, is to be taken as a factor.

The exponent in the expression $6a^2$ applies only to the value represented by a; thus, $6a^2$ equals 6 times the square of a. If the square of both the coefficient 6 and of a were required, the expression would be enclosed in parentheses. When written $(6a)^2$, the parentheses show that the product of 6 and whatever value is represented by a is to be squared. If this expression occurred in a formula, the letter a might represent the length in inches, or some other value, and when using the formula, the numerical value of a would be used, as explained in Chapter III, which deals with the use of formulas.

Roots of Numbers. — The square root of a number is that number which, when multiplied by itself, will give a product equal to the given number. Thus, the square root of 4 is 2, because 2 multiplied by itself gives 4. The square root of 25 is 5; of 36, 6, etc. We may say that the square root is the reverse of the square, so that, if the square of 24 is 576, then the square root of 576 is 24. The mathematical sign for the square root is $\sqrt[2]{\ }$, but the *index figure* (²) is generally left out, making the square-root sign simply $\sqrt{\ }$, thus:

$\sqrt{4} = 2$ (the square root of four equals two),
$\sqrt{100} = 10$ (the square root of one hundred equals ten).

The cube root of a given number is the number which, if repeated as factor three times, would result in the given number. Thus the cube root of 27 is 3, because $3 \times 3 \times 3 = 27$. If the cube of 15 is 3375, then the cube root of 3375 is, of course, 15. The mathematical sign for the cube root is $\sqrt[3]{\ }$, thus:

$\sqrt[3]{64} = 4$ (the cube root of sixty-four equals four),
$\sqrt[3]{4096} = 16$ (the cube root of four thousand ninety-six equals sixteen).

Just as square root is the reverse of square so cube root is the reverse of cube.

In the case of all roots, except the square root, the index, or the small figure in the radical sign (√), must be given. Thus $\sqrt[4]{81}$ represents the fourth root of 81, which root equals 3, since $3 \times 3 \times 3 \times 3 = 81$.

Extracting Square Root. — In solving some shop and tool-room problems, it is necessary to find either the square root or cube root of a number, and ordinarily these values are obtained directly from tables such as are found in practically all engineering handbooks. (See MACHINERY'S HANDBOOK, pages 1 to 41, inclusive.) The use of such tables saves time and insures accurate results. The method of extracting the roots of numbers, however, should be understood.

The operation of finding the square root of a given number is called *extracting* the square root. Assume that the square root of 119,716 is to be found. Write the number as follows, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of two figures each. Should there be an odd number of figures in the given number, the last period to the left will, of course, have only one figure.

11'97'16 | Space for root.

Find the greatest whole number the square of which does not exceed the value of the figures in the left-hand period (11), and write this number as the first figure of the root. In the example this number is 3, the square of which is 9. Subtract this square from the left-hand period, and move down the next period of two figures and annex it to the remainder, thus:

$$\begin{array}{rl} & 11'97'16 \,|\, \underline{3} \\ 3 \times 3 = & \underline{9} \\ & 297 \end{array}$$

Now multiply the figure of the root obtained by the constant 20 which is always used when extracting the square root by this method ($3 \times 20 = 60$), and find how many times this product is contained in the number 297. This gives a trial figure for the second figure of the root; 60 is contained 4 whole times in 297, and 4 is, therefore, placed as the next figure of the root.

$$\begin{array}{rl} & 11'97'16 \;\underline{|\,34} \\ 3 \times 3 = & \underline{9\quad} \\ 3 \times 20 = 60 & 297 \end{array}$$

Now subtract from 297 the product of 60 plus the figure of the root just obtained (4), multiplied by the same figure (4); $(60 + 4) \times 4 = 256$. If this product were larger than 297 it would indicate that the trial figure was too large, and a figure one unit smaller should be used.

Then move down the next period of two figures and annex it to the remainder.

$$\begin{array}{rl} & 11'97'16 \;\underline{|\,34} \\ 3 \times 3 = & \underline{9\quad} \\ 3 \times 20 = 60 & 297 \\ (60 + 4) \times 4 = & \underline{256\quad} \\ & \;4116 \end{array}$$

Now multiply the figures of the root thus far obtained by 20; $(34 \times 20 = 680)$, and find how many times this product is contained in 4116. This gives a trial figure for the third figure of the root; 680 is contained 6 times in 4116, and 6 is, therefore, placed as the third figure of the root. Then subtract from 4116, the product of 680 plus the figure of the root just obtained (6), multiplied by the same figure (6).

$$\begin{array}{rl} & 11'97'16 \;\underline{|\,346} \\ 3 \times 3 = & \underline{9\quad} \\ 3 \times 20 = 60 & 297 \\ (60 + 4) \times 4 = & \underline{256\quad} \\ 34 \times 20 = 680 & \;4116 \\ (680 + 6) \times 6 = & \;\underline{4116} \end{array}$$

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 346 is the exact square root of 119,716.

If there is a remainder when the last period of figures has been moved down, place a decimal point after the figures already obtained in the root, annex two ciphers (00) to the remainder, multiply the number so far obtained in the root by 20, and proceed as before until a sufficient number of decimal places have been obtained to give the root with sufficient accuracy.

Example:

	1'25 | 11.18
$1 \times 1 =$	1
$1 \times 20 = 20$	25
$(20 + 1) \times 1 =$	21
$11 \times 20 = 220$	400
$(220 + 1) \times 1 =$	221
$111 \times 20 = 2220$	17900
$(2220 + 8) \times 8 =$	17824

It will be seen from the calculation that, when multiplying by the constant 20, the decimal point is disregarded, and the figures obtained in the root considered as a whole number. The decimal point must, however, be placed in the root as already explained before annexing the two first ciphers (not in the given number) to the remainder, in order to give a correct value to the root.

Square Root of Decimals. — When extracting the square root of a decimal fraction, or when the square root of a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of two figures each, *beginning at the decimal point*, thus:

2'17'63.56'78'5

If the number of decimal places is not an even number, the period to the right will have only one figure instead of two. By placing a cipher after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

2'17'63.56'78'50

It should be borne in mind that the pointing off of periods of two figures each should always be begun at the decimal point, both for the whole numbers and for the decimals. Thus, for instance, the pointing off in the first line below is correct, while the pointing off in the second line is incorrect:

Correctly pointed off:	0.76'34'5	3'26.75'4
Incorrectly pointed off:	0.7'63'45	32'6.7'54

When extracting the square root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

Example:

$$\begin{array}{rl} & 5.71'21 \mid \underline{2.39} \\ 2\times 2 = & \underline{4} \\ 2\times 20 = 40 & 171 \\ (40+3)\times 3 = & \underline{129} \\ 23\times 20 = 460 & 4221 \\ (460+9)\times 9 = & \underline{4221} \end{array}$$

When it is found that the next figure in the root is a cipher, place it as usual in the root, and move down the next period of two figures, in all other respects following the procedure already explained.

Example:

$$\begin{array}{rl} & 9'12'04 \mid \underline{302} \\ 3\times 3 = & \underline{9} \\ \left.\begin{array}{r} 3\times 20 = 60 \\ 30\times 20 = 600 \end{array}\right\} & 1204 \\ (600+2)\times 2 = & \underline{1204} \end{array}$$

Square Root of Common Fractions. — The square root of a common fraction may be obtained by extracting the square root of both numerator and denominator, thus:

$$\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}.$$

When the terms of the fraction are not perfect squares (squares of whole numbers), it is preferable to change the common fraction to a decimal fraction, and extract the square root of this.

Proof of Square Root. — When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by multiplying the root by itself, in which case the product must equal the number given, of which the square root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation; if this approximate root is multiplied by itself, the product should *very nearly* equal the given number; if not, an error has been made.

Extracting Cube Root. — Assume that the cube root of 80,621,568 is to be found. Write the number as below, leaving space for the figures of the root as shown. Beginning at the unit figure (the last figure at the right of a whole number), point off the number into periods of *three* figures each. According to the total number of figures in the given number, the last period to the left will, of course, have one, two or three figures.

80'621'568 | Space for root.

Now find the greatest whole number, the cube of which does not exceed the value of the figures in the left-hand period (80), and write this number as the first figure in the root. The cube of 4 is 64 ($4 \times 4 \times 4 = 64$), and the cube of 5 is 125 ($5 \times 5 \times 5 = 125$). Hence 4 is the greatest whole number, the cube of which does not exceed 80, and 4, therefore, is the first figure of the root. Subtract the cube of 4 from the left-hand period and move down the next period of three figures, and annex it to the remainder, thus:

80'621'568 | 4
$4 \times 4 \times 4 =$ 64
16621

Now multiply the square of the figure in the root by the constant 300, which is always used when extracting the cube root by this method ($4^2 \times 300 = 4 \times 4 \times 300 = 4800$), and find how many times this product is contained in the number 16,621. This gives us a trial figure for the second figure of the root; 4800 is contained three whole times in 16,621, and 3 is, therefore, placed as the next figure of the root:

$$\begin{array}{rl} & 80'621'568 \ \lfloor 43 \\ 4 \times 4 \times 4 = & \underline{64} \\ 4^2 \times 300 = 4800 & 16621 \end{array}$$

Now subtract from 16,621 the *sum* of the following products:

1. The square of the figure or figures already obtained in the root, excepting the last one, multiplied by 300, and this product multiplied by the figure just obtained in the root, thus:

$$4^2 \times 300 \times 3 = 16 \times 300 \times 3 = 14,400.$$

2. The figure or figures already obtained in the root, excepting the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained, thus:

$$4 \times 30 \times 3^2 = 4 \times 30 \times 9 = 1080.$$

3. The cube of the last figure obtained, thus:

$$3^3 = 3 \times 3 \times 3 = 27.$$

The method followed will be understood by studying the example and comparing the different quantities with the worded explanations just given. If the sum of these various products is larger than 16,621, it indicates that the trial figure is too large, and a figure one unit smaller should be used.

Now move down the next period of three figures, and annex it to the remainder.

$$\begin{array}{rl} & 80'621'568 \ \lfloor 43 \\ 4 \times 4 \times 4 = & \underline{64} \\ 4^2 \times 300 = 4800 & 16621 \\ 4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & \underline{15507} \\ & 1114568 \end{array}$$

Multiply the square of the figures of the root thus far obtained by 300 ($43^2 \times 300 = 43 \times 43 \times 300 = 554,700$), and find how many times this product is contained in 1,114,568. This gives a trial figure for the third figure of the root; 554,700 is contained two times in 1,114,568, and 2 is, therefore, placed as the third figure of the root. Now subtract from 1,114,568 a sum made up of the three products previously given, and shown in the example below:

$$
\begin{array}{rl}
 & 80'621'568 \;\underline{|\;432} \\
4 \times 4 \times 4 = & \underline{64} \\
4^2 \times 300 = 4800 & 16621 \\
4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & \underline{15507} \\
43^2 \times 300 = 554{,}700 & 1114568 \\
43^2 \times 300 \times 2 + 43 \times 30 \times 2^2 + 2^3 = & \underline{1114568}
\end{array}
$$

If, as in the present case, this last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 432 is the exact cube root of 80,621,568.

If there is a remainder when the last period of three figures has been moved down, place a decimal point after the figures already obtained in the root, annex three ciphers (000) to the remainder, multiply the square of the number thus far obtained in the root by 300, and proceed as before until a sufficient number of decimals have been obtained to give the root with sufficient accuracy.

Example:

$$
\begin{array}{rl}
 & 1'816 \;\underline{|\;12.2} \\
1 \times 1 \times 1 = & \underline{1} \\
1^2 \times 300 = 300 & 816 \\
1^2 \times 300 \times 2 + 1 \times 30 \times 2^2 + 2^3 = & \underline{728} \\
12^2 \times 300 = 43{,}200 & 88000 \\
12^2 \times 300 \times 2 + 12 \times 30 \times 2^2 + 2^3 = & \underline{87848}
\end{array}
$$

It should be noted in these calculations that, when squaring the figures thus far obtained in the root and multiplying by the constant 300, the decimal point is disregarded and the figures obtained in the root considered as a whole number. The decimal point, however, must be placed in the root as already explained, before annexing the first three ciphers (not in the given number) to the remainder, in order to give a correct value of the root.

Cube Root of Whole Number and Decimal. — When the cube root of a number containing a whole number and a decimal is required, always point off *both* the whole number and the decimal in periods of three figures each, *beginning at the decimal point*, thus:

83'675'731.563'75

If the number of decimal places is not evenly divisible by three, the period to the right will have only one or two figures instead of three. By placing one or two ciphers after the decimal in such cases, the last period is made complete without changing the value of the number, thus:

83'675'731.563'750

It should be borne in mind that the pointing off of periods of three figures each should always be commenced at the decimal point, both for the whole number and for the decimals. Thus, for instance, the pointing off in the first line below is correct while the pointing off in the second line is incorrect:

Correctly pointed off:	0.765'354'3	2'765.354'2
Incorrectly pointed off:	0.7'653'543	27'65.3'542

Cube Root of Fractions. — In extracting the cube root of a decimal fraction, the decimal point is placed in the root when the first period of decimals is moved down.

When it is found that the next figure in the root is a cipher, place it as usual in the root and move down the next period of three figures, in all other respects following the procedure already explained.

The cube root of a common fraction may be obtained by extracting the cube root of both the numerator and the denominator, thus:

$$\sqrt[3]{\frac{27}{1000}} = \frac{\sqrt[3]{27}}{\sqrt[3]{1000}} = \frac{3}{10}.$$

When the terms of the fraction are not perfect cubes (cubes of whole numbers), it is preferable to change the common fraction to a decimal fraction and then extract the cube root.

Proof of Cube Root. — When there is no remainder after all the periods of figures in the given number have been moved down, and the last figure of the root found, the calculation may be proved by repeating the root as a factor three times, in which case the product must equal the number given, of which the cube root has been extracted. If there is a remainder, the figures obtained do not represent the exact root, but a close approximation. If this approximate root is repeated as a factor three times the product should *very nearly* equal the given number; if not, an error has been made.

Extracting Roots Higher than Square or Cube Roots. — When the root to be extracted is higher than a square or a cube root, the index of the required root is separated into its factors and then the roots indicated by the different factors are extracted successively. For example, the fourth root of 81 ($\sqrt[4]{81}$) is found by first extracting the square root of 81, which equals 9. Then the square root of 9 is extracted to obtain the fourth root of 81. The square root is extracted twice in this case, because the index 4 equals 2×2. That 3 is the fourth root of 81 may be proved as follows: $3 \times 3 \times 3 \times 3 = 81$, the fourth root being repeated as a factor four times.

The sixth root of a number may be obtained by extracting the cube root and then the square root.

Example. — What is the sixth root of 64, or $\sqrt[6]{64}$?

The index 6 equals 3×2 which shows that the cube and square roots should be extracted. The cube root of 64, or $\sqrt[3]{64}$, equals 4, and the square root of 4, or $\sqrt{4}$, equals 2; hence, the sixth root of 64 equals 2. The square root might have been extracted first without affecting the result.

By using tables of squares and cubes, the fifth root of any given number may be accurately found to several places by interpolation, as the fifth power of a number is equal to the product of its square by its cube. For instance, suppose that the fifth root of 7214 is required, accurately, to five places. A moment's inspection of the tables will show that the product of the square and the cube of 6, or $6^2 \times 6^3 = 7776$, is a little too large. Looking into the fifties and inserting the decimal point in the proper place to get the powers of 5.9, it will be found that the fifth power of 5.9, or 5.9^5, equals $5.9^2 \times 5.9^3 = 7149.2$.

The method of obtaining this fifth root will be further explained. By referring to the tables, it will be seen that the square of 59 is 3481; hence, the square of 5.9 is 34.81, the decimal point being moved two places toward the left. The cube of 59 is 205,379; hence, the cube of 5.9 is 205.379, the decimal point being moved three places toward the left. Therefore, the fifth power of 5.9 equals 205.379 × 34.81 = 7149.2, approximately. This number (7149.2), however, is less than 7214, the fifth root of which is required, but, by interpolation, $100 \times \frac{7214 - 7149.2}{7776 - 7149.2} = 10$. Therefore, 5.910 is part of the root. In the same way, 5.91^5 equals $5.91^2 \times 5.91^3 = 7210.03$, which is also smaller than the number (7214) the fifth root of which is required. The fifth power of 5.92 equals $5.92^2 \times 5.92^3 = 7271.25$, which is greater than 7214. This indicates that the fifth root of 7214 may be determined by interpolation, using the fifth power of 5.91, or 7210.03. Thus, $100 \times \frac{7214 - 7210.03}{7271.25 - 7210.03} = 6$. Therefore, 5.9106 is the required root.

A result closer than four or five places is seldom of special value, and the foregoing method is not very tedious. Fourth and sixth roots may also be found by means of the tables in two operations, interpolation usually being necessary.

Cost of a Mixture when Unit Costs are known. — The term "alligation" is applied to certain processes in arithmetic for ascertaining the relations between the proportions and prices of the ingredients of a mixture and the cost of the mixture per unit of weight or volume. The first example referred to illustrates how the price of an alloy per pound is determined when the proportions of the different ingredients and the cost of each ingredient are known.

Example. — An alloy for a lining bearing is composed of 70 pounds of lead at 4 cents per pound, 10 pounds of tin at 30 cents per pound, 17 pounds of antimony at 9 cents per pound, and 3 pounds of copper at 15 cents per pound. What is the cost of the alloy or mixture per pound, not considering the manufacturing cost?

Rule: Multiply the number of pounds of each of the ingredients by its price per pound, add these products together, The symbols or letters used in formulas simply are inserted instead of the actual figures or numerical values which are substituted in the formula for each specific problem that is to be solved. For instance, if the letter S in a formula represents the speed in feet per minute of a revolving or other moving part, when using this particular formula, the figure or numerical value representing the speed is substituted for S. When all the different letters that may be in the formula are replaced by numerical values or numbers, the result required is obtained by simple arithmetical processes. The letters of the alphabet are the symbols commonly used in formulas, and the signs are simply the ordinary signs such as are used for arithmetical calculations with some additional ones that are necessary for special purposes. Letters from the Greek alphabet are often used to designate angles, although in this book letters of the English alphabet have been used instead in most cases.

A Simple Rule and Formula Compared. — The relation between a rule and a corresponding formula will be illustrated. If the speed of a driving pulley and its diameter are known, the speed of the driven pulley may be determined by the following rule:

Rule: Multiply the speed of the driving pulley in revolutions per minute by its diameter, and divide the product by the diameter of the driven pulley, to obtain the speed of the driven pulley.

Now if S = the speed of the driving pulley; D = the diameter of the driving pulley; d = the diameter of the driven pulley; and s = the speed of the driven pulley, then the following formula represents the rule previously given:

$$s = \frac{S \times D}{d}.$$

This formula merely shows that, to obtain the speed (s) of the driven pulley, the speed (S) of the driving pulley must be multiplied by its diameter, and the product divided by the diameter (d) of the driven pulley. It is evident, then, that *the formula* is practically a picture of the rule and enables

Subtract (algebraically) each number from the mean number 2850 and write the results, as shown, in the same rows as the corresponding subtrahends; note that two of these remainders are positive and the other is negative. Multiply these remainders by such numbers (to be found by trial) as will make the sum of the positive numbers at least approximately equal to the negative number. In this case, the multipliers 3, 1, and 4 were selected. The sum of the positive numbers (1122 + 218) is 1340, which nearly equals the negative number (−1336). The sum of the trial multipliers is 8. It is, therefore, assumed that in eight parts of the alloy there are 3 parts of zinc, 1 part of tin, and 4 parts of copper. Expressed in percentages, the alloy contains copper, 50 per cent; zinc, $37\frac{1}{2}$ per cent; and tin, $12\frac{1}{2}$ per cent.

Proof:

$$\frac{0.2476 \times 3 + 0.2632 \times 1 + 0.3184 \times 4}{8} = 0.28495,$$

which is very nearly 0.285, and coincides with it when reduced to three significant figures.

Positive and Negative Quantities. — In order to be able to solve certain shop problems, a working knowledge of the principles of positive and negative numbers or quantities is required. An explanation of the meaning of these expressions, therefore, will be given, together with the rules for calculations with negative numbers, and examples to make the rules thoroughly understood.

On the thermometer scale, as is well known, the graduations extend upward from zero, the degrees being numbered 1, 2, 3, etc. Graduations also extend downward and are numbered in the same way: 1, 2, 3, etc. The degrees on the scale extending upward from the zero point may be called *positive* and preceded by a plus sign, so that, for instance, +5 degrees means 5 degrees above zero. The degrees below zero may be called *negative* and may be preceded by a minus sign, so that −5 degrees means 5 degrees below zero.

The ordinary numbers may also be considered positive and negative in the same way as the graduations on a thermometer scale. When we count 1, 2, 3, etc., we refer to the numbers that are larger than 0 (corresponding to the degrees *above* the zero point), and these numbers are called positive numbers. We can conceive, however, of numbers extending in the other direction of 0; numbers that are, in fact, less than 0 (corresponding to the degrees below the zero point on the thermometer scale). As these numbers must be expressed by the same figures as the positive numbers, they are designated by a minus sign placed before them. For example, -3 means a number that is as much less than, or beyond, 0 in the negative direction as 3 (or, as it might be written, $+3$) is larger than 0 in the positive direction.

A negative value should always be enclosed within parentheses whenever it is written in line with other numbers; for example:

$$17 + (-13) - 3 \times (-0.76).$$

In this example -13 and -0.76 are negative numbers, and by enclosing the whole number, minus sign and all, in parentheses, it is shown that the minus sign is part of the number itself, indicating its negative value.

It must be understood that when we say $7 - 4$, then 4 is not a negative number, although it is preceded by a minus sign. In this case the minus sign is simply the sign of subtraction, indicating that 4 is to be subtracted from 7; but 4 is still a positive number or a number that is larger than 0.

Rules for Adding Negative Numbers. — It now being clearly understood that positive numbers are all ordinary numbers greater than 0, while negative numbers are conceived of as less than 0, and preceded by a minus sign which is a part of the number itself, we can give the following rules for calculations with negative numbers.

Rule: A negative number can be added to a positive number by subtracting its numerical value from the positive number.

Examples:

$$4 + (-3) = 4 - 3 = 1.$$
$$16 + (-7) + (-6) = 16 - 7 - 6 = 3.$$
$$327 + (-0.5) - 212 = 327 - 0.5 - 212 = 114.5.$$

In the last example, 212 is not a negative number, because there are no parentheses indicating that the minus sign is a part of the number itself. The minus sign, then, indicates only that 212 is to be subtracted in the ordinary manner.

As an example illustrating the rule for adding negative numbers to positive ones, the case of a man having $12 in his pocket, but owing $9, may be taken. His debt is a negative quantity, we may say, and equals (-9). Now if he adds his cash and his debts, to find out how much he really has, we have:

$$12 + (-9) = 12 - 9 = 3.$$

Of course, in a simple case like this, it is obvious that 9 would be subtracted directly from 12, but the example serves the purpose of illustrating the method used when a negative number is added to a positive number.

Subtracting Negative Numbers. — *Rule:* A negative number can be subtracted from a positive number by adding its numerical value to the positive number.

Examples:

$$4 - (-3) = 4 + 3 = 7.$$

$$16 - (-7) = 16 + 7 = 23.$$

$$327 - (-0.5) - 212 = 327 + 0.5 - 212 = 115.5.$$

In the last example, note that 212 is subtracted, because the minus sign in front of it does not indicate that 212 is a negative number.

As an illustration of the method used when subtracting a negative number from a positive one, assume that we are required to find how many degrees difference there is between 37 degrees above zero and 24 degrees below; this latter may be written (-24). The difference between the two numbers of degrees mentioned is then:

$$37 - (-24) = 37 + 24 = 61.$$

A little thought makes it obvious that this result is right, and the example shows that the rule given is based on correct reasoning.

Multiplication and Division of Negative Numbers. — *Rule:* When a positive number is multiplied or divided by a negative number, multiply or divide the numerical values as usual; but the product or quotient, respectively, becomes negative. The same rule holds true if a negative number is divided by a positive number.

Examples:

$$4 \times (-3) = -12. \qquad (-3) \times 4 = -12.$$

$$\frac{15}{-3} = -5. \qquad \frac{-15}{3} = -5.$$

Rule: When two negative numbers are multiplied by each other, the product is positive. When a negative number is divided by another negative number the quotient is positive.

Examples:

$$(-4) \times (-3) = 12. \qquad \frac{-4}{-3} = 1.333.$$

When Subtrahend is Larger than Minuend. — If, in subtraction, the number to be subtracted is larger than the number from which it is to be subtracted, the calculation can be carried out by subtracting the smaller number from the larger, and indicating that the remainder is negative.

Examples:

$$3 - 5 = -(5 - 3) = -2.$$

In this example 5 cannot, of course, be subtracted from 3, but the numbers are reversed, 3 being subtracted from 5, and the remainder indicated as being negative by placing a minus sign before it.

$$227 - 375 = -(375 - 227) = -148.$$

The examples given, if carefully studied, will enable the student to carry out calculations with negative numbers when such will be required in solving triangles.

CHAPTER III

WHAT FORMULAS ARE AND HOW THEY ARE USED

FORMULAS are commonly used in books and periodicals dealing with mechanical subjects, because, in many cases, they are preferable to rules; in fact, a formula is practically a rule expressed by signs and symbols or letters instead of using words to describe the order and kind of operations to be performed.

There are two main reasons why a formula is generally preferable to a rule expressed in words. 1. The formula is more concise, it occupies less space, and it is possible for the eye to catch at a glance the whole meaning of the rule laid down. 2. It is easier to remember a short formula than a long rule, and it is, therefore, of greater value and convenience. In this book, rules and the corresponding formulas are given in most cases, and it will be apparent to those who study the different problems that the formula is usually more readily understood.

Some shop men consider formulas difficult, and class all formulas as problems in algebra, because different values are represented by letters or other symbols. Knowledge of algebra is not necessary in order to make possible the successful use of formulas for the solving of problems such as occur in ordinary shop practice. On the contrary, most formulas are solved by ordinary arithmetic and, in many cases, require only addition, subtraction, division, and multiplication. A thorough understanding of the rules and processes of arithmetic is essential. A knowledge of algebra is necessary when a general rule or formula which gives the answer to a problem directly is not available. In such cases, algebra is useful in developing or originating a formula, but the latter can be used without recourse to algebraic processes.

one to see, at a glance, that, to obtain the value of s, it is simply necessary to multiply the values of S and D and divide the product by the value of d.

To illustrate just how this formula is used, suppose the speed represented by S is 150 revolutions per minute; that diameter D equals 40 inches; and diameter d equals 20 inches. Then, when these numerical values are substituted for the different letters, the speed s of the driven pulley, in this particular case, is easily determined. Thus,

$$s = \frac{150 \times 40}{20} = 300.$$

Therefore, the speed of the driven pulley, or the numerical value represented by the letter s, is 300, which is the number of revolutions per minute made by the driven pulley.

Formulas often appear rather complicated to those who are not familiar with them, when, in reality, they are simple and easily solved. To use any formula, simply replace the letters in the formula by all the figures which are given for a certain problem, and find the required answer the same as in arithmetic.

In some of the formulas, two letters represent one quantity or numerical value. For instance, the letters H.P. are often used to represent horsepower, Ng may represent the number of teeth in a gear and Np the number of teeth in a pinion. This practice, however, is not very general, and, as a rule, single letters should be used in preference, in order to avoid confusion and mistakes. The following formula, which may be used for finding the center distance between two meshing spur gears, illustrates the use of the symbols Ng and Np previously referred to.

$$C = \frac{Ng + Np}{2 \times P}.$$

In this formula, C = the center-to-center distance; Ng = the number of teeth in the gear; Np = the number of teeth in the pinion; and P = the diametral pitch of the gear. This formula shows that, to obtain the center distance, the number of teeth in the gear and the pinion must be added and the sum divided by 2 times the diametral pitch.

Omission of Multiplication Signs in Formulas. — The sign for multiplication, or (×), is frequently omitted in formulas. In the following formula for determining the horsepower of a steam engine, H = the indicated horsepower of the engine; P = the mean effective pressure of the piston in pounds per square inch; L = the length of the stroke in feet; A = area of the piston in square inches; and N = the number of strokes made by the piston per minute. Then,

$$H = \frac{P \times L \times A \times N}{33{,}000}.$$

Instead of placing the multiplication signs between the different letters, these are generally omitted in this and other formulas. When the signs are omitted in this particular formula, it is written as follows:

$$H = \frac{PLAN}{33{,}000}.$$

The signs indicating multiplication are not necessary because it is understood by those who are familiar with the use of formulas that the letters representing the numerical values are to be multiplied, and the signs are left out as a matter of convenience. The expression $P \times L \times A \times N$ is just the same as $PLAN$. All of the other signs are indicated the same as in arithmetic. The multiplication sign is never left out between two numbers; thus, 24 always means "twenty-four" and "two times four" must be written 2×4. The expression "two times P," however, may be written $2P$ instead of $2 \times P$. The figure is ordinarily written first in an expression of this kind, and it is known as the "coefficient"; thus, in the expression $2P$, 2 is the coefficient of P. When the letter is written first, the multiplication sign is inserted, as, for example, $P \times 2$. When two letters represent one value, as in the formula previously given for determining the center distance between meshing gears, the symbol Ng does not of course represent $N \times g$, but it is the same as a single letter or symbol, which represents a numerical value.

Why Parentheses are Used in Some Formulas. — When expressions occurring in formulas are enclosed by parentheses

( ) or by brackets [ ], this indicates that the values inside of the parentheses or brackets should be considered as a single value, or that the operations inside the parentheses or brackets should be performed before other calculations. The following simple examples will show just how parentheses are used and how they may affect the result:

Example. — $5 \times 4 - 2 = 20 - 2 = 18$, but, if $4 - 2$ is enclosed by parentheses, this shows that 2 is to be subtracted from 4 before multiplying by 5. Thus, $5 \times (4 - 2) = 5 \times 2 = 10$.

In the following example, two expressions are enclosed by parentheses. Thus, $10 \times (20 + 4) - (3 - 2) = 10 \times 24 - 1 = 239$. In this case, 10 is multiplied by 24 and 1 is then subtracted from the product.

Order of Performing Operations. — Mistakes are often made in solving simple problems because the operations are not performed in the right order. For instance, in simplifying an expression like $10 + 4 \times 5$, the operations are performed frequently in the order written, instead of consideration being given to the manner in which the different values are connected by signs. The general rule is to perform all multiplication first and then the other operations in the order written. Division should also be performed before addition or subtraction when the division is indicated in the same line with the other processes, although this general rule may be modified when parentheses are used, as just explained.

In finding the value of the expression $10 + 4 \times 5$, the 4 and 5 are first multiplied, and the product 20 is added to 10. Hence, the value of the expression is 30. If the operations were performed in the order written, the result would be $14 \times 5 = 70$, instead of 30. The reason why the multiplication should be performed first is that the numbers connected by the multiplication sign are only factors of a product, and should, therefore, be regarded as one number.

That mistakes are often made by performing such operations in succession is indicated by the fact that in a shop containing about 300 men there were only two who were sure that $10 + 4 \times 5$ equals 30. Most of the men obtained the result 70. In order to make this expression equal 70, it would be necessary to put parentheses around the expression $10 + 4$; that is, $(10 + 4) \times 5 = 70$. Otherwise, the multiplication should be carried out before the addition, and the result would be 30.

If we substitute a for 10, b for 4, and c for 5, we have $a + b \times c$, or, as it is commonly written, $a + bc$. A glance at this expression shows at once that b is to be multiplied by c before the result of the multiplication is added to a. If it is required to add a to b, and then multiply by c, the expression would have to take the form $(a + b)c$. If we insert the mathematical values for a, b, and c in these two expressions, we get 30 and 70, respectively.

How Formulas are Transposed. — As shown by the preceding examples, the common method of writing a formula is to place on one side of the equals sign (=) a letter which represents the value or quantity to be determined, and on the opposite side of the equals sign, the letters (or letters and numbers combined) which represent the known values. A formula was previously given for determining the speed (s) of a driven pulley when its diameter (d), and the diameter (D) and speed (S) of the driving pulley are known. The formula is as follows: $s = \frac{S \times D}{d}$. Now, if the speed of the driven pulley is known and the problem is to find its diameter or the value of d instead of s, this formula can be transposed or changed. Thus: $d = \frac{S \times D}{s}$.

It is essential to know how a formula can be changed to determine the values represented by different letters of the formula. Changing a formula in this way is known as "transposition" and the changes are governed by four general rules.

Rule 1. An independent term preceded by a plus sign (+) may be transposed to the other side of the equals sign (=) if the plus sign is changed to a minus sign (−).

Rule 2. An independent term preceded by a minus sign may be transposed to the other side of the equals sign if the minus sign is changed to a plus sign.

As an illustration of these rules, if $A = B - C$, then $C = B - A$, and if $A = C + D - B$, then $B = C + D - A$. That the foregoing is correct may be proved by substituting numerical values for the different letters and then transposing them as shown.

Rule 3. A term which multiplies all the other terms on one side of the equals sign may be transposed to the other side, if it is made to divide all the terms on that side.

As an illustration of this rule, if $A = BCD$, then $\frac{A}{BC} = D$. Suppose, in the preceding formula, that $B = 10$, $C = 5$, and $D = 3$; then $A = 10 \times 5 \times 3 = 150$, and $\frac{150}{10 \times 5} = 3$.

Rule 4. A term which divides all the other terms on one side of the equals sign may be transposed to the other side, if it is made to multiply all the terms on that side.

As an illustration of this rule, if $s = \frac{SD}{d}$, then $sd = SD$, and, according to Rule 3, $d = \frac{SD}{s}$. This formula may also be transposed for determining the values of S and D; thus $\frac{ds}{D} = S$, and $\frac{ds}{S} = D$.

If, in the transposition of formulas, minus signs precede quantities, the signs may be changed to obtain positive rather than minus quantities. All the signs on both sides of the equals sign or on both sides of the equation may be changed. For example, if $-2\,A = -B + C$, then $2\,A = B - C$. The same result would be obtained by placing all the terms on the opposite side of the equals sign which involves changing signs. For instance, if $-2\,A = -B + C$, then $B - C = 2\,A$.

Formula Containing the Power of a Number. — The power of a quantity or number may be given in a formula, and it may be desirable to transpose the formula so that the number itself may be determined. The formula $V = 0.5236d^3$ is for finding the volume of a spherical body. In this formula, V = the volume in cubic inches and d = the diameter of the sphere. Assume that the formula is to be transposed for determining the value of d. $V = 0.5236d^3$; then $d^3 = \frac{V}{0.5236}$. It follows, then, that the cube root of d equals the cube root of $\frac{V}{0.5236}$, or $\sqrt[3]{d^3} = \sqrt[3]{\frac{V}{0.5236}}$. As $d = \sqrt[3]{d^3}$, then $d = \sqrt[3]{\frac{V}{0.5236}}$. If the volume of the sphere is 4.1888 cubic inches, then $d = \sqrt[3]{\frac{4.1888}{0.5236}} = \sqrt[3]{8} = 2$ inches.

Transposition when Formula Requires Extraction of a Root. — The following example illustrates how a formula may be transposed to determine the value of a quantity covered by a root sign.

If A equals the length of a hypotenuse of a right-angled triangle, B equals the altitude, and C equals the length of the base, then $A = \sqrt{B^2 + C^2}$. If this formula is to be transposed for determining the value of C (lengths A and B being known), the first step is to remove the square-root sign, because C^2 cannot be transposed while it is covered by this sign. Now, if A equals $\sqrt{B^2 + C^2}$, it follows that the square of A equals the square of $\sqrt{B^2 + C^2}$, and the square of $\sqrt{B^2 + C^2}$ is the same as $B^2 + C^2$; that is, the square of the expression is obtained by simply removing the square-root sign. The reason why this is true will, perhaps, be clearer if numerical values are substituted for the letters. Suppose $B = 4$ and $C = 3$, then $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$, and the square of $5 = 25$. The sum of $4^2 + 3^2$ also equals 25.

It is evident, then, that $A^2 = B^2 + C^2$. The expression has now been changed so that it can be transposed, the square-root sign having been removed. Thus, $A^2 - B^2 = C^2$, or, if the formula is written in the usual manner with the letter representing the quantity to be determined placed on the left-hand side of the equals sign, $C^2 = A^2 - B^2$. Now, the procedure is the same as for the formula previously referred to for determining the diameter of a spherical body of given volume. Thus, $\sqrt{C^2} = \sqrt{A^2 - B^2}$, and as $C = \sqrt{C^2}$, it follows that $C = \sqrt{A^2 - B^2}$.

CHAPTER IV

HOW TO CALCULATE AREAS OF PLANE SURFACES

In connection with mechanical work, it is frequently necessary to determine the areas of surfaces as well as the volumes of both solid and hollow objects. The area of a surface is expressed in square inches or in square feet. If the pressure in pounds per square inch on the head of a cylinder is known, and the total pressure is required, the area of the surface subjected to the pressure is first determined, and there are a great variety of other problems the solution of which depends in part upon areas. The areas of plane figures will be considered in this chapter, and the volumes of solids in the following chapter.

Squares. — The square has four sides of equal length, and each of the four angles between the sides is a right or 90-degree angle. The area of the square equals the length of the side multiplied by itself, or the square of the length of the side. If the side of a square is 14 inches, then the area equals $14 \times 14 = 196$ square inches. If the side is 14 feet, then the area is 196 square feet.

If the area of a square is known, the length of the side equals the square root of the area. Assume that the area of a square equals 1024 square inches. Then the side equals $\sqrt{1024} = 32$ inches.

Rectangles. — The rectangle has four sides, of which those opposite each other are of equal length, and the four angles between the sides are right or 90-degree angles. The area of a rectangle is found by multiplying the height or altitude by the length or base. If the height is 6 inches and the length, 11 inches, then the area equals $6 \times 11 = 66$ square inches.

If the area of a rectangle and the length of its base are known, the height is found by dividing the area by the length of the known base. Either the longer or the shorter side may be considered as the base, the altitude being the side at right angles to the base. If the area of the rectangle is 96 square inches and the length of the base is 12 inches, then the height equals 96 ÷ 12 = 8 inches.

One square foot equals 12 × 12 = 144 square inches. If the area is given in square feet, it can, therefore, be transformed into square inches by multiplying by 144. If the area is given in square inches, it can be transformed into square feet by dividing by 144.

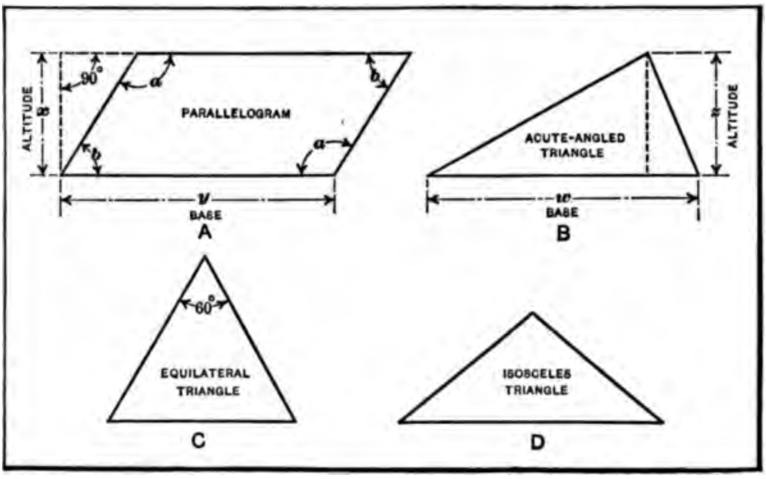


Fig. 1. Parallelogram and Triangles

Parallelograms. — Two lines are said to be parallel when they have the same direction; when extended, they do not meet or intersect, and the same distance is maintained between the two lines at every point. Any figure made up of four sides, of which those opposite are parallel, is called a *parallelogram*. The square and rectangle are parallelograms in which all the angles are right angles. At *A*, Fig. 1, is shown a parallelogram where two of the angles are less and two more than 90 degrees. A line drawn from one side of a parallelogram at right angles to the opposite side is called the height or altitude of the parallelogram. Dimension x is the altitude, and y *is the length* or base.

The area of a parallelogram equals the altitude multiplied by the base. If x is 16 inches, and y, 22 inches, then the area equals $16 \times 22 = 352$ square inches. If the area and the base are given, the altitude is found by dividing the area by the base. In parallelograms the angles opposite each other are alike, as indicated by the fact that the two angles a are equal, and the two angles b also are equal.

Triangles. — Any figure bounded by three straight lines is called a *triangle*. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the height or altitude of the triangle. If the side w of the triangle shown at B, Fig. 1, is taken as the base of the triangle, then z is the altitude.

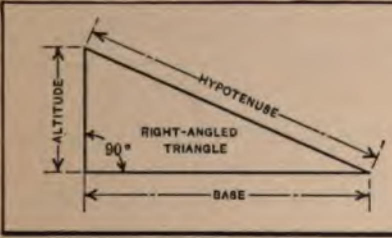


Fig. 2. Right-angled Triangle

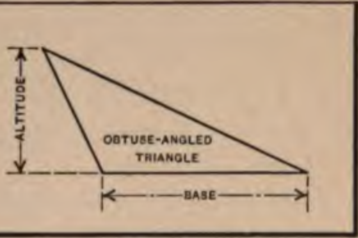


Fig. 3. Obtuse-angled Triangle

If all three sides of a triangle are of equal length, as in the one shown at C, the triangle is called *equilateral*. Each of the three angles in an equilateral triangle equals 60 degrees.

If two sides are of equal length, as shown at D, the triangle is an *isosceles* triangle.

If one angle is a right or 90-degree angle, the triangle is called a *right* or *right-angled* triangle. Such a triangle is shown in Fig. 2; the side opposite the right angle is called the *hypotenuse*.

If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle, as shown as B, Fig. 1. If one of the angles is larger than 90 degrees, as shown in Fig. 3, the triangle is called an *obtuse* or *obtuse-angled* triangle.

The sum of the three angles in every triangle is 180 degrees. The area of any triangle equals one-half the product of the base and the altitude; thus the area of the triangle shown at B in Fig. 1 equals $\frac{1}{2} \times w \times z$. If w equals 9 inches and z, 6 inches, then the area equals $\frac{1}{2} \times 9 \times 6 = 27$ square inches. The area of a triangle may also be found by the following rule: The area of a triangle equals one-half the product of two of its sides multiplied by the sine of the angle between them. The application of this rule is dealt with in Chapter XI. (See the paragraph headed "Areas of Triangles.")

If the area and base of a triangle are known, the altitude can be found by dividing twice the area by the length of the base. If the area and the altitude are known, the base is found by dividing twice the area by the altitude. If the area

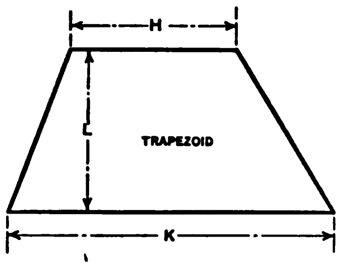


Fig. 4. Trapezoid

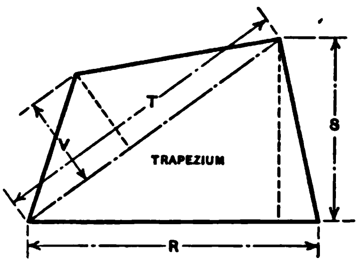


Fig. 5. Trapezium

of a triangle is 180 square inches, and the base is 18 inches, then the altitude equals $(2 \times 180) \div 18 = 20$ inches.

Trapezoids. — When a figure is bounded by four lines, of which only two are parallel, it is called a *trapezoid*. The height of a trapezoid is the distance L, Fig. 4, between the two parallel lines H and K. The area of a trapezoid equals one-half the sum of the lengths of the parallel sides multiplied by the height. The area of the trapezoid in Fig. 4 thus equals $\frac{1}{2} \times (H + K) \times L$. If $H = 16$ feet, $K = 24$ feet, and $L = 14$ feet, then the area $= \frac{1}{2}(16 + 24) \times 14 = 280$ square feet.

Trapeziums. — When a figure is bounded by four lines, no two of which are parallel, as shown in Fig. 5, it is called a *trapezium*. The area of a trapezium is found by dividing it into two triangles as indicated by the dash-and-dot line, finding the area of each of the two triangles, and adding these areas. The dotted lines indicate the altitudes of the two triangles into which the trapezium has been divided. If the dimensions of the base and height of the one triangle are R and S, respectively, and of the other, T and V, as shown, then the area of the whole trapezium would be $(\frac{1}{2} \times R \times S) + (\frac{1}{2} \times T \times V)$. Assume that $R = 20$ feet, $S = 17$ feet, $T = 23$ feet, and $V = 9$ feet, then the area of the trapezium $= (\frac{1}{2} \times 20 \times 17) + (\frac{1}{2} \times 23 \times 9) = 273.5$ square feet.

The Circle. — If the diameter of a circle is known, the circumference is found by multiplying the diameter by 3.1416. (See Fig. 6 for meaning of terms.) Assume that the circumference of a circle is stretched out into a straight line by the circle rolling upon a flat surface and unfolding itself, then the length of the straight line would be three times the diameter plus a distance equal to 0.1416 times the diameter; or the whole length of the circumference would be 3.1416 times the diameter. As the diameter equals 2 × radius, the circumference equals 2 × radius × 3.1416.

If the circumference of a circle is known, the diameter is found by dividing the circumference by 3.1416; the radius is found by dividing the circumference by 2 × 3.1416. Instead of writing out the number 3.1416, the Greek letter π (pi) is often used; thus, for example, $3\pi = 3 \times 3.1416$.

The area of a circle equals the square of the radius multiplied by 3.1416; or the square of the diameter multiplied by 0.7854.

If the area of a circle is known, the radius is found by extracting the square root of the quotient of the area divided by 3.1416.

If D = diameter, R = radius, A = area, then:

$$A = R^2 \times 3.1416;$$

$$A = \frac{D^2 \times 3.1416}{4} = D^2 \times 0.7854;$$

$$R = \sqrt{\frac{A}{3.1416}}.$$

Examples. — The diameter of a circle is 6 inches, find the area.

Using the formula given, we have:

Area = $6^2 \times 0.7854 = 6 \times 6 \times 0.7854 = 28.2744$ square inches.

The area of a circle is 95.033 square inches, find the radius.

Using the formula given, we have:

Radius = $\sqrt{95.033 \div 3.1416} = 5.5$ inches.

Circular Sectors. — A figure bounded by a part of the circumference of a circle and two radii is called a circular *sector*. (See Fig. 6.) The angle b between the radii is called the angle

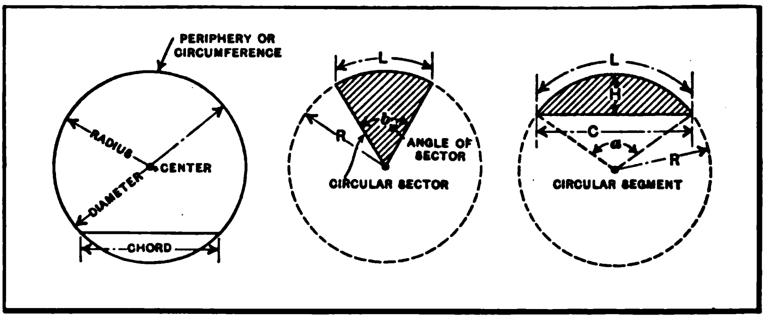


Fig. 6. Circle, Circular Sector, and Circular Segment

of the sector, and the length L of the circumference of the circle is called the arc of the sector.

If R = radius of circle of which the sector is a part;
b = angle of sector, in degrees;
L = length of arc of sector;
A = area of sector;

then the formulas below are used:

$$L = \frac{R \times b \times 3.1416}{180} = \frac{2 \times A}{R};$$

$$b = \frac{180 \times L}{R \times 3.1416};$$

$$A = \frac{L \times R}{2};$$

$$R = \frac{2 \times A}{L} = \frac{180 \times L}{b \times 3.1416}.$$

If the radius of a circle is $1\frac{1}{2}$ inch, and the angle of a circular sector is 60 degrees, how long is the arc of the sector?

Using the given formula, we have:

$$L = \frac{1\frac{1}{2} \times 60 \times 3.1416}{180} = 1.5708 \text{ inch.}$$

What is the area of the same sector?

From the formula given, we have:

$$A = \frac{1.5708 \times 1\frac{1}{2}}{2} = 1.1781 \text{ square inch.}$$

Circular Segments. — A figure bounded by a part of the circumference of a circle and a chord is called a circular *segment*. (See Fig. 6.) The distance H from the chord to the highest point of the circular arc is called the height of the segment.

If R = radius;
C = length of chord;
L = length of arc of segment;
H = height of segment;
A = area of segment;
then the following formulas are used:

$$C = 2 \times \sqrt{H \times (2 \times R - H)};$$

$$R = \frac{C^2 + 4 \times H^2}{8 \times H};$$

$$A = \frac{L \times R - C \times (R - H)}{2}.$$

If the angle a is given, instead of the length of arc L, the length of the arc is found by the formula:

$$L = \frac{R \times a \times 3.1416}{180}.$$

Assume that the radius of a segment is 5 feet and the height, 8 inches. How long is the chord of this segment?

First transform 5 feet into inches; $5 \times 12 = 60$ inches. Then apply the formula given:

$$C = 2 \times \sqrt{8 \times (2 \times 60 - 8)} = 2 \times \sqrt{896} = 2 \times 29.93 = 59.86 \text{ inches.}$$

The length of the chord of a segment is 16 inches and the height, 6 inches. How long is the radius of the circle of which the segment is a part?

Applying the formula given:

$$R = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = 8\tfrac{1}{3} \text{ inches.}$$

Regular Polygons. — Any plane surface or figure bounded by straight lines is called a *polygon*. If all the sides are of equal length and the angles between the sides are equal, the figure is called a *regular* polygon. A regular polygon having five sides is shown at *A* in Fig. 7. The five sides are of the same length *S* and the angles *b* are equal.

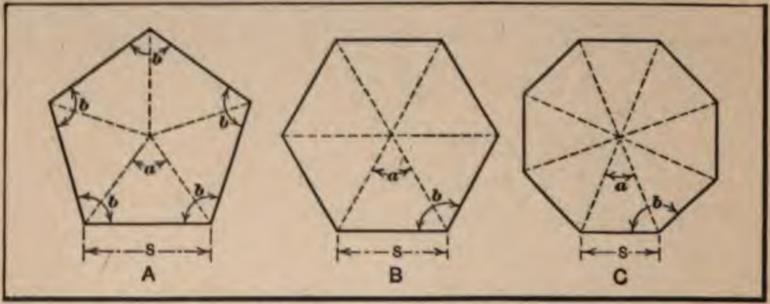


Fig. 7. Regular Polygons

A regular polygon with five sides is called a *pentagon;* one with six sides (as at *B*), a *hexagon;* one with seven sides, a *heptagon;* and one with eight sides (as at *C*), an *octagon*. When a regular polygon has only three sides (Fig. 8), it becomes an equilateral triangle, and when it has four sides, a square.

A circle may be drawn so that it passes through all the angle-points of a regular polygon, as shown in Figs. 8 and 9; such a circle (with the radius *R*) is said to be *circumscribed* about the polygon. The smaller circle in the same illustrations (with the radius *r*) which touches or is tangent to the sides of the polygon, is said to be *inscribed* in the polygon. The centers of the circumscribed and inscribed circles are located at the same point. If the angle-points of the polygon are connected by lines with this center, as shown by the dotted lines in Fig. 7, the polygon is divided up into a number of triangles of equal size and shape. The number of triangles equals the number of sides in the polygon.

The angle *a* of each of these triangles at the center can be determined for any polygon when the number of sides is known. This angle, in degrees, equals 360 divided by the number of sides in the regular polygon, or, expressed as a formula, if *N* equals the number of sides:

$$\text{Angle } a = \frac{360}{N}.$$

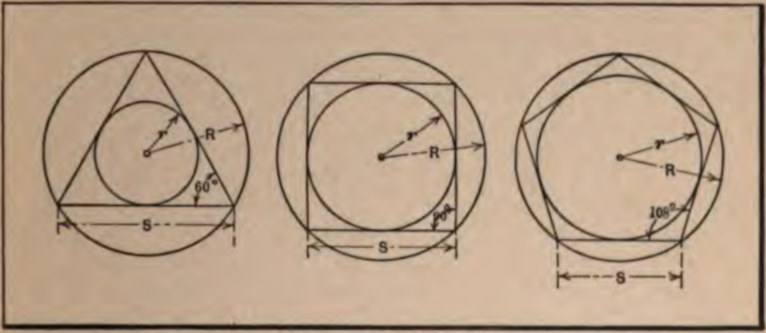


Fig. 8. Equilateral Triangle, Square, and Pentagon

The angle *b* between two adjacent sides of the polygon (see Fig. 7) equals *a* subtracted from 180, or:

$$\text{Angle } b = 180 - a.$$

The area of a polygon can be found by dividing it into triangles, as shown in Fig. 7. After having measured the base and height of one triangle and calculated its area, the area of the whole polygon is found by multiplying the area of one triangle by the number of triangles or sides.

For the more commonly used regular polygons, the following formulas give the area directly when the length of the side is known.

Equilateral Triangles: Since the sum of the three angles in any triangle equals 180 degrees, each of the angles in an equilateral triangle equals $\frac{1}{3}$ of 180 degrees, or 60 degrees.

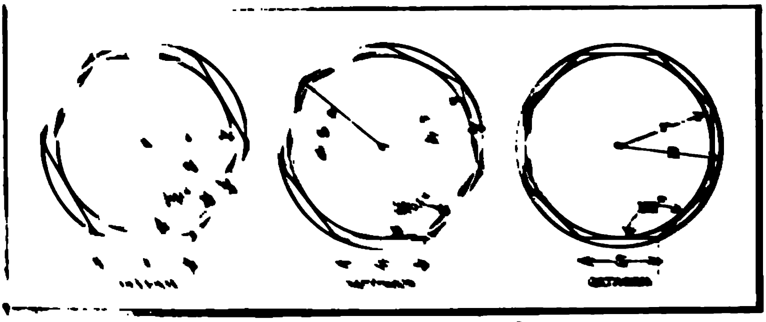
The radius *r* of the circle inscribed in an equilateral triangle (see Fig. 8) equals the side multiplied by 0.289.

The radius *R* of the circumscribed circle equals the side multiplied by 0.577.

If the radius of the circumscribed circle is known, the side is found by multiplying the radius by 1.732.

If the radius of the inscribed circle is known, the side is found by multiplying the radius by 3.464.

The area of an equilateral triangle equals the square of the side multiplied by 0.433; or, the square of the radius of the circumscribed circle multiplied by 1.299; or, the square of the radius of the inscribed circle multiplied by 5.196.



[illegible]

[illegible]

[illegible] formulas as follows:

[illegible]

[illegible] (see Fig. 9) [illegible] equals the side multi- [illegible]

The side of a square equals twice the radius of the inscribed circle, or 1.414 times the radius of the circumscribed circle.

The area equals the square of the side. The area also equals the square of the radius of the circumscribed circle multiplied by 2; or, the square of the radius of the inscribed circle multiplied by 4.

Using the same meaning for the letters as before, the previous rules may be expressed in formulas as follows:

$$r = 0.5 \times S;$$
$$R = 0.707 \times S;$$
$$S = 1.414 \times R = 2 \times r;$$
$$A = S^2 = 2 \times R^2 = 4 \times r^2.$$

The Pentagon: In the pentagon (see diagram A, Fig. 7) the angle b between the sides equals 108 degrees. This is found by the formulas previously given as shown below:

$$N = \text{number of sides} = 5;$$
$$a = \frac{360}{N} = \frac{360}{5} = 72 \text{ degrees};$$
$$b = 180 - a = 180 - 72 = 108 \text{ degrees}.$$

The following formulas are used for finding the radii of the circumscribed and inscribed circles, the side and the area of regular pentagons:

$$r = 0.688 \times S;$$
$$R = 0.851 \times S;$$
$$S = 1.176 \times R = 1.453 \times r;$$
$$A = 1.720 \times S^2 = 2.378 \times R^2 = 3.633 \times r^2.$$

The Hexagon: In the hexagon (see Fig. 9) the length of the side S equals the radius R of the circumscribed circle so that each of the six triangles formed when lines are drawn from the center to the angle-points, are equilateral triangles. The angle between two adjacent sides equals the sum of two angles in two of the equilateral triangles and, consequently, equals 60 + 60 = 120 degrees.

Using the same letters as previously given in the formulas, we have for the hexagon:

$r = 0.866 \times S;$
$R = S;$
$S = R = 1.155 \times r;$
$A = 2.598 \times S^2 = 2.598 \times R^2 = 3.464 \times r^2.$

The Heptagon: The heptagon (see Fig. 9) has seven sides, and the angle between two adjacent sides is found as follows:

N = number of sides = 7;

$$\text{Angle } a = \frac{360}{N} = \frac{360}{7} = 51\frac{3}{7} \text{ degrees;}$$

$$\text{Angle between adjacent sides} = 180 - 51\frac{3}{7} = 128\frac{4}{7} \text{ degrees.}$$

Using the same letters as in the formulas previously given, we have for the heptagon:

$r = 1.038 \times S;$
$R = 1.152 \times S;$
$S = 0.868 \times R = 0.963 \times r;$
$A = 3.634 \times S^2 = 2.736 \times R^2 = 3.371 \times r^2.$

The Octagon: The angle between two adjacent sides of the octagon, as shown in Fig. 9, is 135 degrees.

Using the same meaning for the letters as previously given, the formulas for the octagon are:

$r = 1.207 \times S;$
$R = 1.307 \times S;$
$S = 0.765 \times R = 0.828 \times r;$
$A = 4.828 S^2 = 2.828 \times R^2 = 3.314 \times r^2.$

Practical Examples Involving Areas. — It is often necessary to determine the area of some surface, as, for example, when a surface is subjected to a certain pressure, and it is essential to obtain the total pressure, or the pressure per square inch when the total pressure is known.

Example. — The diameter of the plunger of a hydraulic press is 10 inches, and it is subjected to a pressure of 550 pounds per square inch. What is the total pressure on the plunger?

As the area of a circle equals the square of the diameter multiplied by 0.7854, the area of a 10-inch plunger equals $10^2 \times 0.7854 = 78.54$ square inches; hence, the total pressure equals $78.54 \times 550 = 43{,}197$ pounds, or 21.6 tons, nearly.

Example. — The total pressure against a piston is 3800 pounds and the piston is 4 inches in diameter. What is the pressure in pounds per square inch?

The area of the piston equals $4^2 \times 0.7854 = 12.56$ square inches. Therefore, the pressure per square inch equals $3800 \div 12.56 = 302$ pounds per square inch, approximately.

Example. — If a structural steel bar $\frac{1}{2}$ inch square sustains a steady load of 2950 pounds, what is the stress in pounds per square inch, and is this a safe load?

The area of the bar equals $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ square inch, and the stress in pounds per square inch equals $2950 \div \frac{1}{4} = \frac{2950}{1} \times \frac{4}{1} = 11{,}800$ pounds per square inch.

As the average ultimate tensile strength of structural steel is about 60,000 pounds per square inch, the $\frac{1}{2}$-inch square bar will safely sustain this load, since it is only about one-fifth of the load that would be required to break the bar.

CHAPTER V

HOW TO CALCULATE VOLUMES, WEIGHTS, AND CAPACITIES

CALCULATIONS relating to volumes may be necessary not only to determine the volume of a solid or hollow object, but also as a means of comparing the volumes or sizes of solid bodies or hollow receptacles of different proportions. Volumes are also determined when estimating how much a part made of a given material will weigh, as, for example, when figuring the weights of castings when only the drawings are available. The capacities of hollow objects such as tanks or other receptacles are determined by first finding the volume. For instance, if the diameter and length (or height) of a cylindrical tank are known, and the problem is to determine how many gallons it will hold, the capacity in gallons can be determined readily if the volume is known. Volume is expressed either in cubic inches or in cubic feet.

Volume of a Cube. — The cube (Fig. 1) is a solid body having six surfaces or faces, all of which are squares; as all the faces are squares, all the sides are of equal length. If the side of a face of a cube equals S, the volume equals $S \times S \times S$ or, as it is commonly written, S^3.

Assume that the length of the side of a cube equals 3 inches; then the volume equals $3 \times 3 \times 3 = 27$ cubic inches.

When the volume of a cube is known, the length of the side is found by extracting the cube root of the volume.

Assume that the volume of a cube equals 343 cubic inches. If we extract the cube root of 343, we find that the side of the cube is 7 inches.

One cubic foot equals $12 \times 12 \times 12 = 1728$ cubic inches; therefore, a volume given in cubic feet can be transformed into cubic inches by multiplying by 1728; if the volume is given in cubic inches it can be transformed into cubic feet by dividing by 1728.

Volume of Prisms. — A solid body, the sides of which are all rectangles, and the ends of which are either rectangles or squares, is commonly called a *square prism*. Opposite surfaces or faces are parallel, and all the angles are right angles. A square prism is shown in Fig. 2, where L is its length, W, its width, and H, its height. The volume of a square prism equals the length times the width times the height, or, expressed as a formula, if V = volume,

$$V = L \times W \times H.$$

Assume that $L = 20$ inches, $W = 4$ inches, and $H = 5$ inches, then volume $= 20 \times 4 \times 5 = 400$ cubic inches.

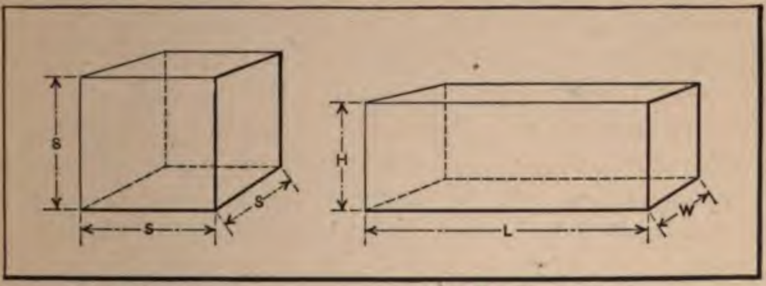


Fig. 1. Cube **Fig. 2. Square Prism**

A solid body having the end faces parallel, and the lines along which the other faces intersect or meet parallel, is called a *prism*. The two parallel end faces are called *bases*. The length, height, or altitude L, Fig. 3, of a prism is the distance between the bases, measured at right angles to the base surfaces.

The volume of a prism equals the area of the base multiplied by the length or height of the prism. The area of the base must, therefore, first be found before the volume can be obtained. If the base is a triangle, parallelogram, trapezoid, trapezium, or a regular polygon, its area is found by the rules previously given for such plane figures. If it is a polygon that is not regular, it can always be divided into triangles, and the area of each of the triangles can be calculated, and these areas added together to obtain the area of the whole polygon.

Assume that it is required to find the volume of a prism, the base of which is a regular hexagon having a side S; the length of the prism is L. The volume of this prism is:

$$2.598 \times S^2 \times L.$$

If, in this example, S equals $1\frac{1}{2}$ inch, and L equals 9 inches, then the volume equals:

$2.598 \times 1\frac{1}{2}^2 \times 9 = 2.598 \times 1.5 \times 1.5 \times 9 = 52.6095$ cubic inches.

Volume of a Pyramid. — A solid body having a polygon for the base and a number of triangles all having a common

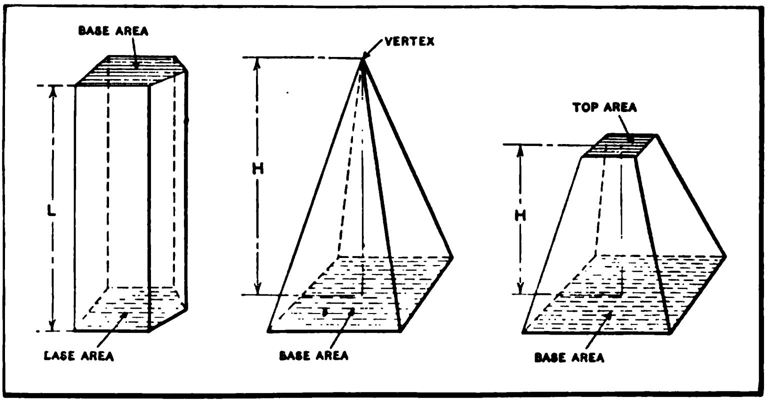


Fig. 3. Prism **Fig. 4. Pyramid** **Fig. 5. Frustum of Pyramid**

vertex for the sides is called a *pyramid*. In Fig. 4, a pyramid is shown in which the base has four sides and the side surfaces are made up of triangles having two equal sides. If a line is drawn from the vertex of the pyramid at right angles to the base, the length of this line is the altitude or height H of the pyramid.

The volume of a pyramid equals the base area multiplied by one-third of the height. It is, therefore, necessary to find the base area before the volume can be found.

Assume that it is required to find the volume of a pyramid, the base of which is a regular pentagon, having a side S; the height of the pyramid is H. The volume of the pyramid equals:

$$1.720 \times S^2 \times \tfrac{1}{3} \times H \text{ (area of base} \times \text{one-third the height).}$$

If $S = 2$ inches and $H = 9$ inches, then the volume equals $1.720 \times 2^2 \times \frac{1}{3} \times 9 = 1.720 \times 2 \times 2 \times 3 = 20.640$ cubic inches.

A *frustum of a pyramid* is shown in Fig. 5. It is a pyramid from which the top has been cut, the top surface being parallel to the base. The height of a frustum of a pyramid is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a pyramid can be found when the height, the top area, and the base area are known.

If V = volume of frustum of a pyramid;
H = height of frustum;
A_1 = area of top;
A_2 = area of base;
then

$$V = \frac{H}{3} \times (A_1 + A_2 + \sqrt{A_1 \times A_2}).$$

Assume, for example, that the base of a frustum of a pyramid is a square, and that the side of the square is 5 inches. The top area is, of course, also a square; assume the side of this to be 2 inches. The height of the frustum is 6 inches. By first calculating the base and top areas and then inserting the values in the formula given, the volume is obtained.

$$\begin{aligned} \text{Volume} &= \frac{6}{3} \times (5^2 + 2^2 + \sqrt{5^2 \times 2^2}) \\ &= 2 \times (25 + 4 + \sqrt{25 \times 4}) \\ &= 2 \times (25 + 4 + 10) = 78. \end{aligned}$$

The Prismoidal Formula. — The prismoidal formula is a general formula by which the volume of any prism, pyramid, or frustum of a pyramid, and the volume of any solid body bounded by regular curved surfaces may be found.

If A_1 = area at one end of the body;
A_2 = area at other end;
A_m = area of a middle section between the two end surfaces;
H = height of the body;
V = volume of body;

then

$$V = \frac{H}{6} \times (A_1 + 4\,A_m + A_2).$$

As this formula applies to all regular solid bodies, it is useful to remember. For ordinary calculations, however, the formulas previously given for each kind of solid should be used because of greater simplicity.

Volume of a Cylinder. — A solid body having circular and parallel end faces of equal size is called a *cylinder*. (See Fig. 6.) The two parallel faces are called *bases*. The height or altitude H of a cylinder is the distance between the bases measured at right angles to the base surfaces.

The volume of a cylinder equals the area of the base multiplied by the height. The area of the base, therefore, must be

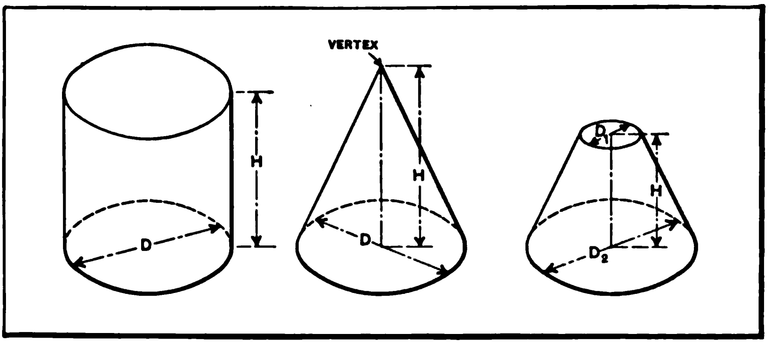


Fig. 6. Cylinder Fig. 7. Cone Fig. 8. Frustum of Cone

found before the volume can be obtained. If the diameter of the base is D, the area of the base equals $0.7854\,D^2$. The volume of the cylinder then equals:

$$0.7854 \times D^2 \times H.$$

If $D = 3$ inches and $H = 5$ inches, then the volume equals $0.7854 \times 3^2 \times 5 = 0.7854 \times 3 \times 3 \times 5 = 35.343$ cubic inches.

Volume of a Cone. — A solid body having a circular base and the sides inclined so that they meet at a common vertex, the same as in a pyramid, is called a *cone*. (See Fig. 7.) If a *line is drawn from* the vertex of the cone at right angle to the base, the length of this line is the altitude or height H of the cone.

The volume of a cone equals the base area multiplied by one-third of the height. It is necessary, therefore, to find the area of the base circle before the volume can be found. If the diameter of the base equals D, then the area equals $0.7854\ D^2$, and this multiplied by one-third of the height H gives us the volume:

$$0.7854 \times D^2 \times \tfrac{1}{3} \times H = \tfrac{1}{3} \times 0.7854 \times D^2 \times H = 0.2618 \times D^2 \times H.$$

If the diameter of the base of a cone equals 4 inches and the height, 6 inches, then the volume equals:

$$0.2618 \times 4^2 \times 6 = 0.2618 \times 4 \times 4 \times 6 = 25.1328 \text{ cubic inches.}$$

A frustum of a cone is shown in Fig. 8. It is a cone from which the top has been cut, the top surface being a circle parallel to the base. The height H of a frustum of a cone is the length of a line drawn from the top surface at right angles to the base.

The volume of a frustum of a cone can be found when the diameters of the top and base circles, and the height are known.

If V = volume of frustum of a cone;
H = height of frustum;
D_1 = diameter of top circle;
D_2 = diameter of base circle;

then

$$V = 0.2618 \times H \times (D_1^2 + D_2^2 + [D_1 \times D_2]).$$

Assume, for example, that the diameter of the base of a frustum of a cone is 5 inches, and that the diameter of the top circle is 2 inches. The height of the frustum is 6 inches. By inserting these values in the formula given, we have:

$$V = 0.2618 \times 6 \times (2^2 + 5^2 + [2 \times 5])$$
$$= 0.2618 \times 6 \times (4 + 25 + 10)$$
$$= 0.2618 \times 6 \times 39 = 61.2612 \text{ cubic inches.}$$

Volume of a Sphere. — The name "sphere" is applied to a solid body shaped like a ball or globe, that is, bounded by a surface which at all points is at the same distance from a point inside of the sphere called its *center*. The diameter of a sphere is the length of a line drawn from a point on the surface through the center to the opposite side. (See Fig. 9.)

The volume of a sphere equals 3.1416 multiplied by four-thirds of the cube of the radius, or 3.1416 multiplied by one-sixth of the cube of the diameter.

If R = radius of the sphere, D = diameter, and V = volume, this rule can be written as formulas thus:

$$V = 3.1416 \times \tfrac{4}{3} \times R^3 = 4.1888 \times R^3;$$
$$V = 3.1416 \times \tfrac{1}{6} \times D^3 = 0.5236 \times D^3.$$

If the volume of a sphere is known, the radius can be found by extracting the cube root of the quotient of the volume divided by 4.1888; the diameter can be found by extracting the cube root of the quotient of the volume divided by 0.5236.

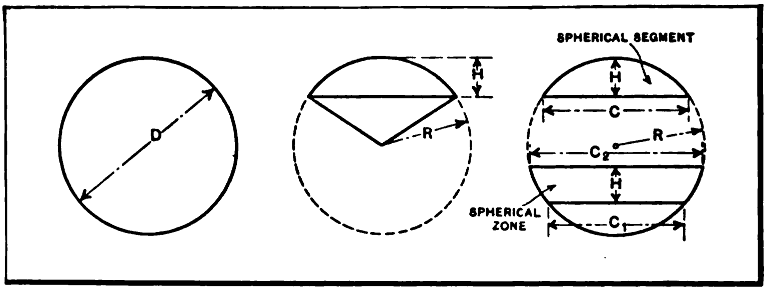


Fig. 9. Sphere Fig. 10. Spherical Sector Fig. 11. Spherical Segment and Zone

Written as formulas, these rules are:

$$R = \sqrt[3]{\frac{V}{4.1888}}; \qquad D = \sqrt[3]{\frac{V}{0.5236}}.$$

Volume of Spherical Sector and Segment. — A *spherical sector* is a part of a sphere bounded by a section of the spherical surface and a cone, having its vertex at the center of the sphere, as shown in Fig. 10. The volume of a spherical sector can be found if the radius R and the height H are known.

The formula for the volume V is:

$$V = 2.0944 \times R^2 \times H.$$

Assume that the length of the radius of a spherical sector is 15 inches and the height is 4 inches. Then the volume equals:

$$2.0944 \times 15^2 \times 4 = 2.0944 \times 15 \times 15 \times 4 = 1884.96 \text{ cubic inches.}$$

A *spherical segment* is a part of a sphere bounded by a portion of the spherical surface and a plane circular base, as shown in Fig. 11. The volume of a spherical segment can be found when the radius of the sphere and the height H of the segment, or the diameter C of the base of the segment and its height H, are known.

If V = volume of segment;
H = height of segment;
R = radius of the sphere of which the segment is a part;
C = diameter of the base of the segment;

then

$$V = 3.1416 \times H^2 \times \left(R - \frac{H}{3}\right)$$

$$V = 3.1416 \times H \times \left(\frac{C^2}{8} + \frac{H^2}{6}\right)$$

Assume that the height of a spherical segment is 6 inches and the radius, 8 inches, then the volume is:

$$3.1416 \times 6^2 \times (8 - 6 \div 3) = 3.1416 \times 6 \times 6 \times (8 - 2)$$
$$= 3.1416 \times 6 \times 6 \times 6 = 678.5856 \text{ cubic inches.}$$

Volume of Spherical Zone. — A *spherical zone* is bounded by a part of a spherical surface, and by two parallel circular bases, as shown in Fig. 11, where C_1 and C_2 are the diameters of the circular bases of the zone, and H its height.

The volume of a spherical zone can be found when the height of the segment and the two base diameters are known.

If V = volume of zone;
C_1 = diameter of the smaller base circle;
C_2 = diameter of the larger base circle;
H = height of zone;

then

$$V = 0.5236 \times H \times \left(\frac{3\,C_1^2}{4} + \frac{3\,C_2^2}{4} + H^2\right).$$

Assume that the diameter $C_1 = 3$ inches, the diameter $C_2 = 4$ inches, and the height of the segment equals 1 inch, then the volume is

$$0.5236 \times 1 \times \left(\frac{3 \times 3^2}{4} + \frac{3 \times 4^2}{4} + 1^2\right) =$$

$$0.5236 \times 1 \times \left(\frac{27}{4} + \frac{48}{4} + 1\right) = 0.5236 \times 1 \times 19.75 = 10.3411$$

cubic inches

[If a plane parallel with the end faces and passing through the center of the sphere intersects the zone, consider the zone as two zones, one zone being on each side of the center. Calculate the volume of each, and add these to find the total volume.]

Dimensions of a Rectangular Area in the Same Ratio as the Sides of a Given Rectangle. — To find the dimensions of a rectangular area that shall have the same ratio between the sides as a given smaller rectangle, divide the area of the required rectangle by the area of the given rectangle, and extract the square root of the quotient. The square root is the factor by which the dimensions of the given rectangle are to be multiplied to yield the dimensions of the required rectangle. For example, if a rectangular steel plate measures 3 by 4 feet, what are the dimensions of a plate having 192 square feet, with the sides of the same ratio?

The area of the first plate mentioned is $3 \times 4 = 12$ square feet. 192 feet divided by 12 equals 16. The square root of 16 is 4. Multiplying both dimensions of the 3- by 4-foot plate by 4 gives 12 and 16. $12 \times 16 = 192$ square feet, the required rectangle.

The same procedure is followed for a solid as in the case of a rectangle, except that the cube root of the ratio of the given and required solids is found, and dimensions of the given solid are multiplied by the cube root, the result being the dimensions of the required solid.

Example. — A tank is 3 by 4 by 5 feet, and it is desired to construct another tank containing 480 cubic feet with sides *in the same ratio.* What are the dimensions? Divide 480 by 60 (the cubic contents of the given tank), extract the cube root of the quotient, and the root is 2. Then the required tank dimensions will be 6 by 8 by 10 feet.

Specific Gravity and Weights of Materials. — The expression "specific gravity" indicates how many times a certain volume of a material is heavier than an equal volume of water. If it is found, for example, that one cubic inch of steel weighs 7.8 times as much as one cubic inch of pure water, the specific gravity of steel is 7.8.

As the density of water differs slightly at different temperatures, it is usual to make comparisons on the basis that the water has a temperature of 62 degrees F. The weight of one cubic inch of pure water at 62 degrees F. is 0.0361 pound. If the specific gravity of any material is known, the weight of a cubic inch of the material can, therefore, be found by multiplying its specific gravity by 0.0361.

The specific gravity of cast iron, for example, is 7.2. The weight of one cubic inch of cast iron is found by multiplying 7.2 by 0.0361. The product, 0.260, is the weight of one cubic inch of cast iron.

As there are $12 \times 12 \times 12 = 1728$ cubic inches in one cubic foot, the weight of a cubic foot is found by multiplying 1728 by the weight of a cubic inch.

If the weight of a cubic inch of a material is known, the specific gravity is found by dividing the weight per cubic inch by 0.0361.

The weight of a cubic inch of gold is 0.6975 pound. The specific gravity of gold is then found by dividing 0.6975 by 0.0361. The quotient, 19.32, is the specific gravity of gold.

If the weight per cubic inch of any material is known, the weight of any volume of the material is found by multiplying the weight per cubic inch by the volume expressed in cubic inches. If brass weighs 0.289 pound per cubic inch, 16 cubic inches of brass, of course, weigh $0.289 \times 16 = 4.624$ pounds. In an example of this kind, if the specific gravity is known, instead of the weight per cubic inch, this latter weight is first found by the rule previously given for finding the weight per cubic inch from the specific gravity.

If the specific gravity of tool steel is 7.85, what is the weight of 12 cubic inches of tool steel? The weight of one cubic inch is found by multiplying 7.85 by 0.0361. The product, 0.283, is then multiplied by 12 to find the weight of 12 cubic inches; $0.283 \times 12 = 3.396$ pounds.

Estimating Weight of Bar Stock. — The weight of a piece of round bar stock can be found by first calculating the volume of the piece. When the volume is found in cubic inches, the weight is found by multiplying the volume by the weight of the material per cubic inch, as already explained.

If the diameter of a piece of round tool steel bar is 2 inches, and the length is 7 inches, the volume of this piece equals $0.7854 \times$ square of diameter $\times$ length, or $0.7854 \times 2^2 \times 7 = 21.991$ cubic inches. The volume in cubic inches having been found, it is multiplied by the weight of tool steel per cubic inch, which is 0.283 pound. The weight of the bar is then $21.991 \times 0.283 = 6.2235$ pounds. The specific gravities and weights per cubic inch of various metals and alloys will be found in engineering handbooks (see "Specific Gravity" in index of MACHINERY'S HANDBOOK).

In order to find the weight of a hexagonal bar, when the width across flats, the length, and the weight per cubic inch of the material from which the bar is made, are known, the area of its end section must first be found so that the volume can be determined by multiplying this area by the length; when the width across flats is given, this area equals 0.866 times the square of the width across flats.

Assume that the weight is to be found of a hexagonal piece of machine steel bar stock 3 inches across flats, and 6 inches long. The volume of this piece equals, then, $0.866 \times 3^2 \times 6 = 0.866 \times 3 \times 3 \times 6 = 46.764$ cubic inches, and the weight equals $46.764 \times 0.283 = 13.234$ pounds. The factor 0.283 is the weight of one cubic inch of machine steel.

Estimating the Weight of Castings. — The weight of a *casting* can be calculated when the volume of the casting and the specific gravity or the weight per cubic inch of the material from which the casting is made, are known. If the volume is known in cubic inches, the volume is simply multiplied by the weight per cubic inch to obtain the weight of the casting.

The specific gravity of cast iron is 7.2 and the weight per cubic inch is 0.260; the specific gravity of brass is 8 and the weight per cubic inch is 0.289.

The problem of finding the weight of castings is chiefly one of finding the volume of the casting. The multiplication by the weight per cubic inch of the material is then a simple matter.

Assume that it is required to find the weight of a hollow cast-iron cylinder, as shown in Fig. 12, where the outside

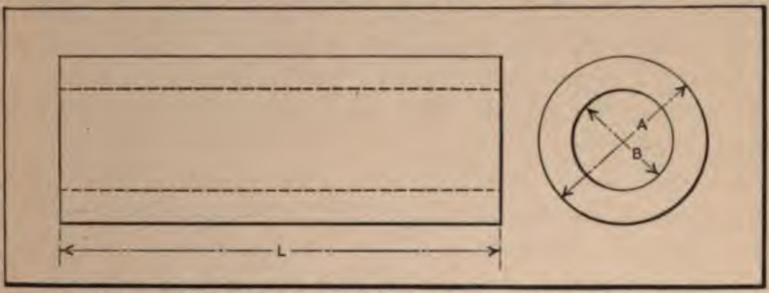


Fig. 12. Hollow Cylinder, the Weight of Which is to be estimated

diameter is A, the inside or core diameter B, and the length L. To find the volume, first calculate the volume of a cylinder with the diameter A and the length L; then subtract from this the volume of the cylinder forming the core.

Assume that $A = 3$ inches, $B = 2$ inches, and $L = 8$ inches. The volume of a cylinder = 0.7854 × the square of the diameter × the height. The volume of a cylinder with 3 inches diameter and a height of 8 inches $= 0.7854 \times 3^2 \times 8 = 56.5488$ cubic inches. From this is subtracted the volume of the cylinder forming the core, which has a diameter of 2 inches. The volume of this cylinder is $0.7854 \times 2^2 \times 8 = 25.1328$ cubic inches. The volume of the hollow cylinder equals $56.5488 - 25.1328 = 31.416$ cubic inches. As the weight per cubic inch of cast iron is 0.260 pound, the total weight of the hollow cylinder will be $31.416 \times 0.260 = 8.168$ pounds.

If the outside diameter of a hollow cylinder is A, the inside diameter B, and the length L, the following formula may be used for finding the volume of the cylinder:

$$\text{Volume} = 0.7854 \times (A^2 - B^2) \times L.$$

Sectional Method of Determining Volume of a Casting. — In Fig. 13 is shown a knee made from cast iron, all the necessary dimensions for calculating the weight being given. To calculate the volume of a casting of this shape, it is divided into

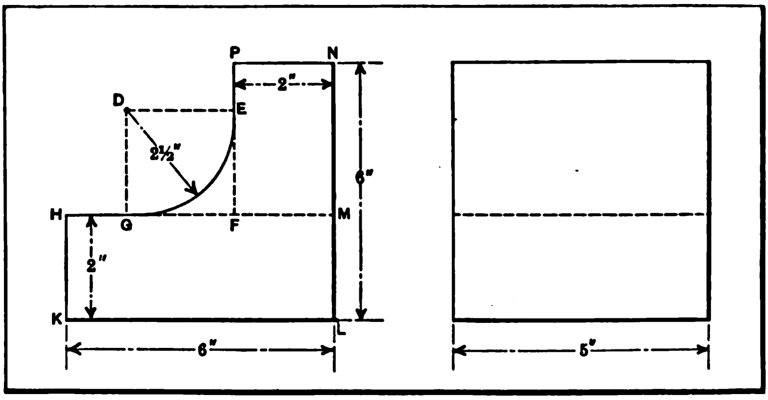


Fig. 13. Bracket or Knee — Another Example illustrating Method of Estimating Weight

prisms or other simple geometric shapes, and the volume of each of the parts is found, after which these volumes are added together to find the total volume of the casting. The piece shown in Fig. 13 can be divided into three parts, the volume of each of which can be calculated by simple means. One part has for base the rectangle $HMLK$, another the rectangle $PFMN$, and the base of the third is bounded by two straight lines EF and FG, and the circular arc EG. The length of all the parts in this case equals the length of the casting, or 5 inches, as shown.

The area of the rectangle $HMLK$ equals $6 \times 2 = 12$ square inches. This area multiplied by 5 equals the volume of this part in cubic inches; $12 \times 5 = 60$ cubic inches.

The length of the line NM is 4 inches ($6 - 2 = 4$), and, therefore, the area of the rectangle *PFMN* is $4 \times 2 = 8$ square inches and $8 \times 5 = 40$ cubic inches.

It now remains to find the volume of the section having for base the area bounded by the two straight lines *EF* and *FG* and the circular arc *EG*. The area of the square *DEFG* is first found and then the area of the circular sector *DEG* is subtracted. The area of the square is $2\frac{1}{2} \times 2\frac{1}{2} = 6\frac{1}{4}$ square inches. The area of the circular sector which is one-fourth of a complete circle is $\frac{2\frac{1}{2}^2 \times 3.1416}{4} = 4.909$ square inches. This subtracted from the area of the square equals 1.341 square inch $(6.25 - 4.909 = 1.341)$. This is the area of the third

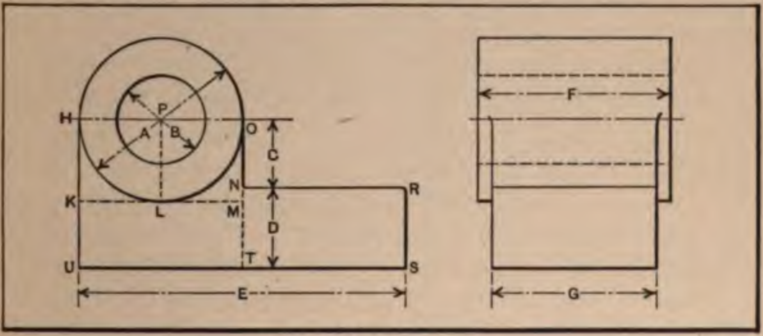


Fig. 14. Bearing Bracket

section into which the casting is divided, and this area multiplied by 5 gives the volume of the third part of the casting $(1.341 \times 5 = 6.705)$. Now adding the volumes of the three parts together, we have $60 + 40 + 6.705 = 106.705$ cubic inches. This total volume multiplied by the weight per cubic inch of cast iron gives the total weight: $106.705 \times 0.260 = 27.743$ pounds. The same method of procedure may be applied to castings of various shapes.

Assume that the weight of a cast-iron bracket, as shown in Fig. 14, is required. All the required dimensions are here given by the letters *A*, *B*, *C*, *D*, *E*, *F*, and *G*. The casting is divided into sections, and the volume of each section is calculated separately; then the volumes are added together and the total volume multiplied by the weight per cubic inch of cast iron. Very small fillets, like those shown at *N* and *R*, are not considered, and the area *NRST* is regarded as a perfect rectangle.

In the example given, the casting is divided into five parts; one is a hollow cylinder with an outside diameter *A*; two parts have for bases the rectangles *NRST* and *KMTU*; and two parts have for bases the areas *HKL* and *OML*, respectively, each being bounded by two straight lines and a circular arc.

For an example, assume that, in Fig. 14, $A = 7$ inches; $B = 4$ inches; $C = 3$ inches; $D = 4$ inches; $E = 12$ inches; $F = 10$ inches; and $G = 8$ inches.

The volumes of the different parts will then be found as follows:

Volume of hollow cylinder having an outside diameter of 7 inches, and inside diameter of 4 inches, and length of 10 inches:

$$0.7854 \times (7^2 - 4^2) \times 10 = 0.7854 \times (49 - 16) \times 10$$
$$= 0.7854 \times 33 \times 10 = 259.18 \text{ cubic inches.}$$

Volume of section having for base the rectangle *NRST*:

$$4 \times 5 \times 8 = 160 \text{ cubic inches.}$$

Volume of section having for base the rectangle *KMTU*:

$$3\tfrac{1}{2} \times 7 \times 8 = 196 \text{ cubic inches.}$$

Volume of section having for base the area *HKL*:

$$\left(3\tfrac{1}{2} \times 3\tfrac{1}{2} - \frac{3\tfrac{1}{2}^2 \times 3.1416}{4}\right) \times 8 = (12.25 - 9.62) \times 8$$
$$= 2.63 \times 8 = 21.04 \text{ cubic inches.}$$

The volume of the section having for base *OML* equals the volume of the section having for base *HKL* and is, consequently, 21.04 cubic inches.

The total of the five sections then equals:

$$259.18 + 160 + 196 + 21.04 + 21.04 = 657.26 \text{ cubic inches.}$$

The total weight of the casting equals $657.26 \times 0.260 = 170.89$ pounds.

When the pattern for a casting contains no core-prints, but is in all respects an exact duplicate of the casting to be made, the weight of the casting may be found approximately by multiplying the weight of the pattern by a constant which varies for different kinds of woods used for the pattern. When the pattern is made from white pine, multiply the weight of the pattern by 13 to obtain the weight of a cast-iron casting; if the pattern is made from cherry, multiply by 10.7; if made of mahogany, multiply by 10.28. When an aluminum pattern is used, the weight of the aluminum pattern may be multiplied by 2.88 to obtain the weight of a cast-iron casting.

Capacity of a Tank in Gallons. — In order to determine the capacity of a tank or other receptacle, the volume in either cubic feet or cubic inches is first determined, and this volume is divided by the number of cubic feet or cubic inches in a U. S. gallon.

Rule: To obtain the capacity of a tank in U. S. gallons, divide the volume of the tank in cubic inches by 231, or the volume of the tank in cubic feet by 1.337.

Example. — If a cylindrical tank is 10 feet long and 3 feet in diameter, how many gallons will it hold?

As the volume of a cylinder equals the area of the base multiplied by the length, the volume in this case equals $3^2 \times 0.7854 \times 10 = 7.068 \times 10 = 70.68$ cubic feet. As one gallon contains 1.337 cubic foot, the capacity of this tank equals $70.68 \div 1.337 = 52$ gallons, approximately.

CHAPTER VI

FIGURING TAPERS

In all circular or round pieces of work, the expressions "taper per inch" and "taper per foot" mean the taper on the diameter, or the difference between the smaller and the larger diameter of a piece, measured one inch or one foot apart, as the case may be. Suppose that the diameter at one end of the tapering part shown at *A* in Fig. 1 is one inch, and the diameter at the other end, one and one-half inch, and that the length of the part is 12 inches, or one foot. This piece, then, tapers one-half inch per foot, because the difference between the diameters at the ends is one-half inch. The diameters at the ends of the part shown at *B* are $\frac{7}{16}$ inch and $\frac{1}{2}$ inch, and the length is one inch; this piece, therefore, tapers $\frac{1}{16}$ inch per inch. Tapers may also be expressed for other lengths than one inch and one foot. For example, the piece shown at *C* tapers $\frac{5}{32}$ inch in 5 inches, the difference between $1\frac{9}{32}$ and $1\frac{1}{8}$ being $\frac{5}{32}$ inch.

If the taper in a certain number of inches is known, the taper in 1 inch can easily be found. If the taper in 5 inches is $\frac{5}{32}$ inch, the taper in 1 inch equals the taper in 5 inches divided by 5, or, in this case, $\frac{5}{32} \div 5 = \frac{1}{32}$, which is the taper per inch. The taper per foot is found by multiplying the taper per inch by 12. In this case, the taper per foot equals $12 \times \frac{1}{32} = \frac{3}{8}$ inch. The length of the work is always measured parallel to the center line (axis) of the work, and never along the tapered surface.

The problems met with in regard to figuring tapers may be of three classes. In the first place, the figures for the large and the small ends of a piece of work may be given, and the length of the work, as at *E*, the problem being to find the taper per foot. In the second place, the diameter at one end may be known, the length of the work, and the taper per foot, as at *D*, the problem being to find the diameter at the other end of the work. In the third place, the required diameters at both ends of the work may be known, and the taper per foot, as at *G*, the problem being to find the dimension between the given diameters, or the length of the piece. Each of these problems will now be dealt with.

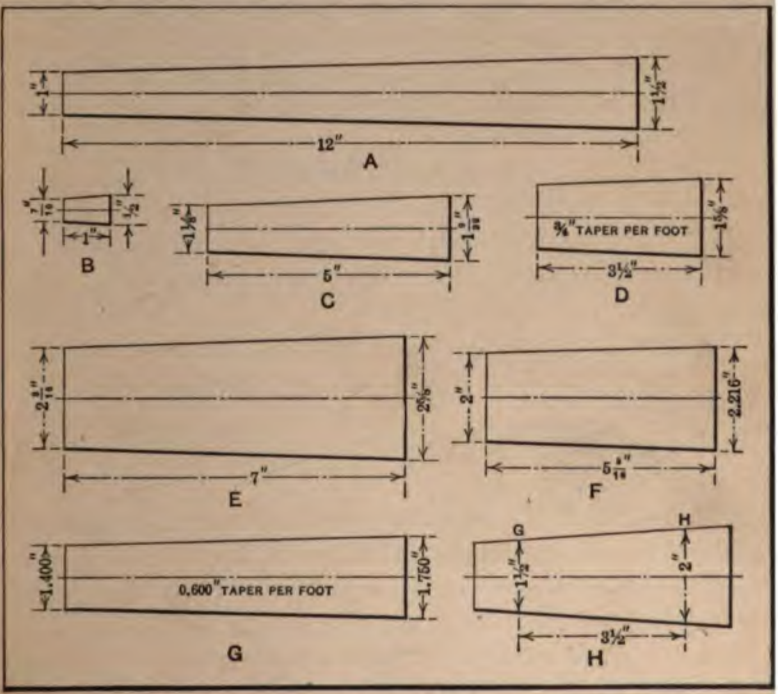


Fig. 1. Miscellaneous Illustrations of Problems in Figuring Tapers

To Find the Taper per Foot when End Diameters and Length are given. — When the diameters at the large and the small ends of the tapering part and the length of the taper are given, the taper per foot may be found by subtracting the small diameter from the large, dividing the difference by the length of the taper, and multiplying the result by 12. The diameter at the large end of the part shown at *E*, Fig. 1, is $2\frac{5}{8}$ inches, the diameter at the small end, $2\frac{3}{16}$ inches, and th length of the work, 7 inches. The taper in 7 inches is then equal to the difference between $2\frac{5}{8}$ inches and $2\frac{3}{16}$ inches, or $\frac{7}{16}$ inch. The taper in one inch equals $\frac{7}{16}$ divided by 7, or $\frac{1}{16}$ inch; and the taper per foot is 12 times the taper per inch, or 12 times $\frac{1}{16}$, which equals $\frac{3}{4}$ inch. The taper per foot, then, equals $\frac{3}{4}$ inch.

If the length is not expressed in even inches, but is $5\frac{3}{16}$ inches, for instance, as at *F*, the procedure is exactly the same. Here the diameter at the large end is 2.216 inches and at the small end, 2 inches. The taper in $5\frac{3}{16}$ inches is, therefore, 0.216 inch. This is divided by $5\frac{3}{16}$ to find the taper per inch.

$$0.216 \div 5\frac{3}{16} = 0.216 \div \frac{83}{16} = 0.216 \times \frac{16}{83} = 0.0416.$$

The taper per inch, consequently, equals 0.0416 inch, and the taper per foot is 12 times this amount, or $\frac{1}{2}$ inch, almost exactly.

Expressed as a formula, if all dimensions given are in *inches*, the previous calculation would take this form:

$$\text{Taper per foot} = \frac{\text{large diameter} - \text{small diameter}}{\text{length of work}} \times 12.$$

It makes no difference if the large and small diameters are measured at the extreme ends of the work or at some other place on the work, provided the length or distance between the points where the diameters are given, is stated. At *H*, Fig. 1, the smaller and larger diameters are given at certain distances from the ends of the work, but the distance ($3\frac{1}{2}$ inches) between these points is given, and the calculation is exactly the same as if the work were no longer than $3\frac{1}{2}$ inches. The following examples will tend to show how the figuring of the taper per foot enters into actual shop work.

Example 1. — The blank for a taper reamer is shown at *A*, Fig. 2. The diameters at the large and small ends of the flutes, and the length of the fluted part, are indicated on the drawing. It is required to find the taper per foot in order to be able to set the taper-turning attachment of the lathe.

Referring to the dimensions given, the difference in diameters at the large and small ends of the taper is $\frac{15}{64}$ inch. This divided by the length of the body, or $7\frac{1}{2}$ inches, gives $\frac{1}{32}$ as the taper per inch. The taper per foot is 12 times the taper per inch, or, in this case, $\frac{3}{8}$ inch. The taper attachment of the lathe is, therefore, set to the $\frac{3}{8}$-inch graduation, and the taper turned will be according to the diameters given on the drawing.

Example 2. — The taper clamping bolt shown at *B*, Fig. 2, is part of a special machine tool. The drawing calls for a diameter of $2\frac{7}{8}$ inches a certain distance from the large end of the

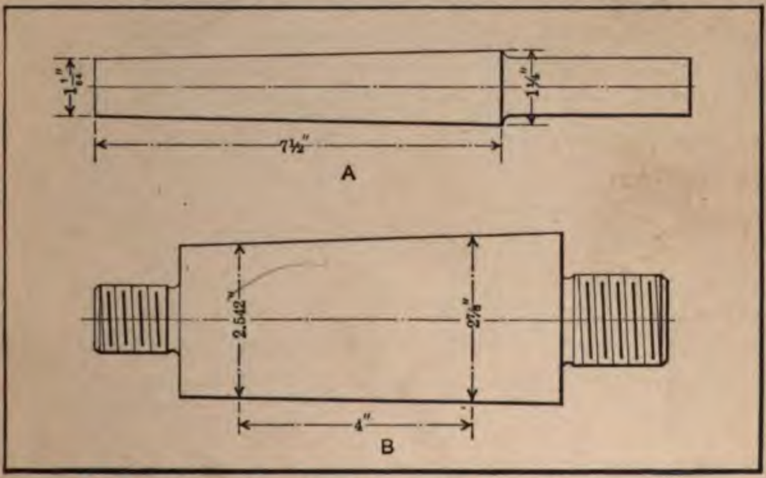


Fig. 2. (*A*) **Blank for Taper Reamer.** (*B*) **Taper Clamping Bolt**

taper, and for a diameter of 2.542 inches a distance 4 inches further down on the taper. The taper in 4 inches is then $2\frac{7}{8}$ inches minus 2.542 inches, or 0.333 inch. The taper in one inch equals 0.333 divided by 4, or 0.0833. The taper per foot is 12 times the taper per inch, or 12 times 0.0833, which equals one inch, almost exactly. The taper to which to turn the bolt is thus one inch per foot.

To Find One Diameter when the Other Diameter, Length, and Taper per Foot are given. — When one diameter, the length of the taper, and the taper per foot are given, the other diameter is found as follows: Divide the taper per foot by 12; multiply the product by the length of the taper, and subtract the result from the large diameter to find the small diameter, or, add the result to the small diameter to find the large diameter.

Referring to sketch *D*, Fig. 1, the diameter at the large end of the work is $1\frac{5}{8}$ inch, the length of the work is $3\frac{1}{2}$ inches, and the taper per foot is $\frac{3}{4}$ inch. The problem is to find the diameter at the small end. In this case we simply reverse the method employed in the previous problems, where it was required to find the taper per foot. In this case, we know that the taper per foot is equal to $\frac{3}{4}$ inch. The taper in one inch must be one-twelfth of this, or $\frac{3}{4}$ inch divided by 12, which equals $\frac{1}{16}$ inch. Now, the taper in $3\frac{1}{2}$ inches, which we want to find in order to know what the diameter is at the small end of the work, must be $3\frac{1}{2}$ times the taper in *one* inch, or $3\frac{1}{2}$ times $\frac{1}{16}$, which equals $\frac{7}{32}$. The taper in $3\frac{1}{2}$ inches, then, is $\frac{7}{32}$ inch, which means that the diameter at the small end of a piece of work, $3\frac{1}{2}$ inches long, is $\frac{7}{32}$ inch smaller than the diameter at the large end. The diameter at the large end, according to our drawing, is $1\frac{5}{8}$ inch. The diameter at the small end, being $\frac{7}{32}$ inch smaller, is, therefore, $1\frac{13}{32}$ inch.

Expressed as a formula, the previous calculation would take this form:

$$\text{Diameter at small end} = \text{Diameter at large end} - \left(\frac{\text{taper per foot}}{12} \times \text{length of taper}\right).$$

Now take a case where the diameter at the small end is given, as at *A*, Fig. 3, and the diameter at the large end is wanted. The figuring is exactly the same, except, of course, the amount of taper in the length of the work is *added* to the small diameter to find the large diameter. When the large diameter is given, the amount of taper in the length of the work is *subtracted* to find the small diameter.

Referring again to sketch *A*, Fig. 3, where the small diameter is given as 1.636 inch, the length of the work as 5 inches, and the taper per foot as $\frac{1}{2}$ inch, how large is the large diameter?

If the taper per foot is ¼ inch, the taper per inch is ¼ divided by 12 which equals 0.0208, and the taper in 5 inches equals 5 times 0.0208, or 0.104 inch. The diameter at the large end of the work, then, is 0.104 inch larger than the diameter at the small end. The diameter at the small end is given on

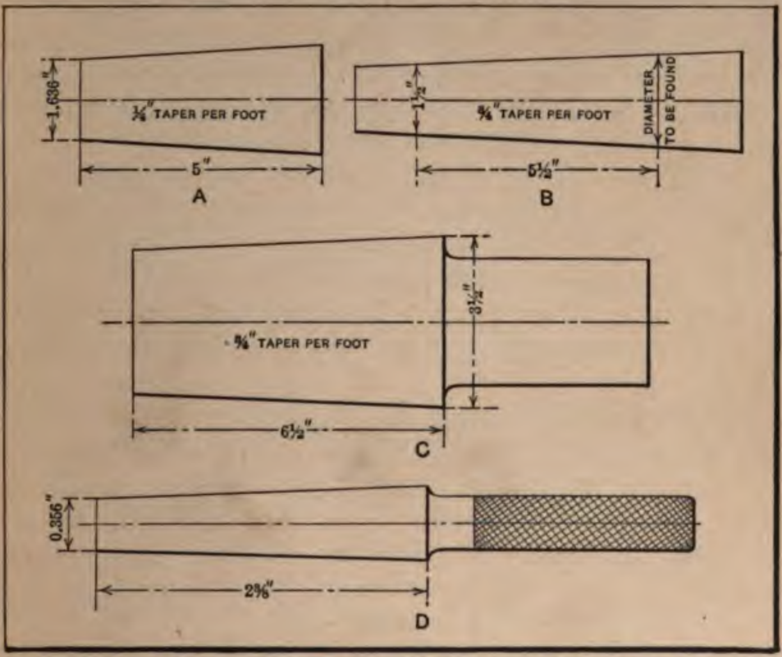


Fig. 3. Finding One Diameter when the Other Diameter, Length, and Taper per Foot are given

the drawing as 1.636 inch; adding 0.104 inch to this, we obtain 1.740 inch as the diameter at the large end.

Expressed as a formula, the previous calculation would take this form:

Diameter at large end =

$$\text{Diameter at small end} + \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right).$$

It may again be well to call attention to the fact that it makes no difference whether the large and small diameters are figured at the extreme ends of the work or at some other points, as long as the diameter to be found is located at one end of the length dimension, and the diameter given on the drawing, at the other. Thus, at *B* in Fig. 3, the diameter is given a certain distance up on the taper, and the diameter which is required is not at the end of the taper. But the dimension $5\frac{1}{2}$ inches is given between the points where these diameters are to be measured, and in figuring one may reason as if the work ended at these points. The following examples, which are of direct practical application to shop work, will prove helpful in remembering the principles outlined.

Example 1. — Sketch *C*, Fig. 3, shows a taper tap, the blank for which is to be turned. The diameter at the large end of the threaded part is $3\frac{1}{2}$ inches, as given on the drawing, the length of the thread is $6\frac{1}{2}$ inches, and the taper per foot is $\frac{3}{4}$ inch. It is required to find the diameter at the small end, in order to measure this end and ascertain that the tap blank has been correctly turned.

The taper per foot being $\frac{3}{4}$ inch, the taper per inch is $\frac{3}{4}$ divided by 12, or $\frac{1}{16}$ inch. The taper in $6\frac{1}{2}$ inches is $6\frac{1}{2}$ times the taper in one inch, or $6\frac{1}{2}$ times $\frac{1}{16}$ inch, which equals $\frac{13}{32}$ inch. The taper in $6\frac{1}{2}$ inches being $\frac{13}{32}$ inch means that the diameter at the small end of the tap blank is $\frac{13}{32}$ inch smaller than the diameter at the large end. The diameter at the small end is, therefore, $3\frac{3}{32}$ inches.

Example 2. — Sketch *D*, Fig. 3, shows a taper gage for a standard Morse taper No. 1. The diameter at the small end is 0.356 inch, the length of the gage part is $2\frac{3}{8}$ inches, and the taper per foot, 0.600 inch. We want the diameter at the large end, in the first place, in order to know what size stock to use for the gage, and later for measuring this diameter, when turned, to see that the taper turned is correct.

A taper of 0.600 inch per foot gives a taper of 0.050 per inch. In $2\frac{3}{8}$ inches, the taper equals $2\frac{3}{8}$ times 0.050, or 0.119 inch. This added to the diameter at the small end gives the diameter at the large end: $0.356 + 0.119 = 0.475$ inch.

Example 3. — Sketch *A*, Fig. 4, shows a taper bolt used as *a clamp bolt*. The diameter, $3\frac{1}{4}$ inches, is given 3 inches from the large end of the taper. The total length of the taper is 10 inches. The taper is $\frac{3}{8}$ inch per foot. It is desired to find the diameters at the extreme large and small ends of this piece.

First the diameter at the large end is found. The taper per foot being $\frac{3}{8}$ inch, the taper per inch equals $\frac{1}{32}$ inch. The taper in 3 inches is, consequently, $\frac{3}{32}$. This added to $3\frac{1}{4}$ inches will give the diameter at the large end, which is $3\frac{11}{32}$ inches.

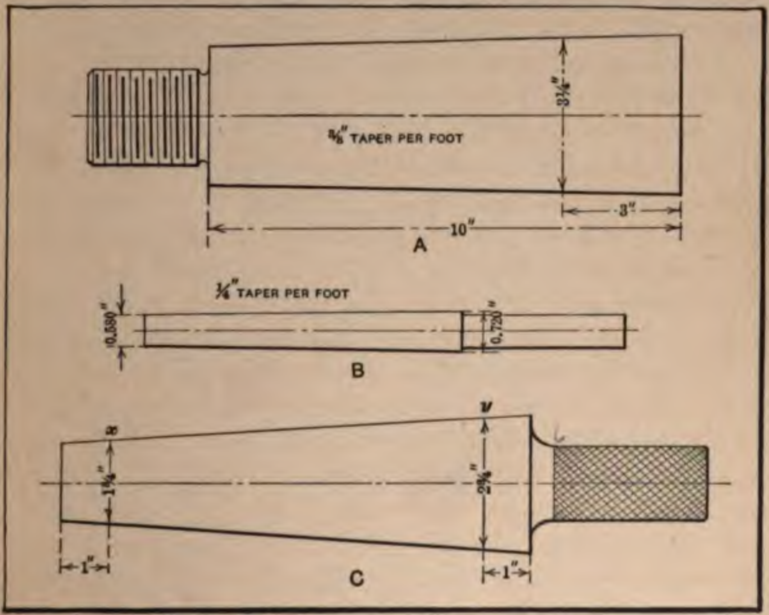


Fig. 4. Additional Problems in Figuring Tapers

To find the diameter at the small end, subtract the taper in 10 inches, which is 10 times the taper in one inch, or 10 times $\frac{1}{32}$, which equals $\frac{5}{16}$, from the diameter $3\frac{11}{32}$ inches at the large end. This gives a diameter at the small end of $3\frac{1}{32}$ inches.

The diameter at the small end can also be found without previously finding the diameter at the extreme large end. The total length of the taper is 10 inches, and the dimension from where the diameter $3\frac{1}{4}$ inches is given to the large end is 3 inches. Consequently, the dimension from where the diameter $3\frac{1}{4}$ inches is given, to the small end, is 7 inches. The taper in one inch is $\frac{1}{32}$ inch; in 7 inches, therefore, it is $\frac{7}{32}$ inch. The diameter at the small end of the work is $\frac{7}{32}$ inch smaller than $3\frac{1}{4}$ inches, or $3\frac{1}{32}$ inches, the same as found previously when we figured from the extreme large diameter of the taper.

To Find the Distance between Two Given Diameters when the Taper per Foot is known. — To find the dimension between two given diameters of a piece of work, when the taper per foot is given, subtract the diameter at the small end from the diameter at the large end, and divide the remainder by the taper per foot divided by 12.

Assume that the diameter at the large end of the piece is 1.750 inch, at the small end, 1.400 inch, and the taper per foot is 0.600 inch. How long is this piece of work required to be, in order to have the given diameters at the ends, with the taper stated? We know that the taper per foot is 0.600 inch. The taper per inch is then 0.600 divided by 12, or 0.050 inch. The difference in diameters between the large and the small ends of the work is 1.750 − 1.400, or 0.350 inch, which represents the taper in the length of the work. Now, we know that the taper is 0.050 inch in *one* inch. How many inches does it then require to obtain a taper of 0.350 inch? This is found by seeing how many times 0.050 is contained in 0.350, or, in other words, by dividing 0.350 by 0.050, which gives 7 as the result. This means that it takes 7 inches for a piece of work to taper 0.350 inch, if the taper is 0.600 per foot. The length of the work, consequently, is 7 inches in the case referred to.

Expressed as a formula, the previous calculation would take the form:

$$\text{Length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per foot} \div 12}.$$

The taper per foot divided by 12, as given in the formula above, of course simply represents the taper per inch. The formula may, therefore, be written:

$$\text{Length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per inch}}.$$

A few examples of the application of these rules will make their use in actual shop work clearer.

Example 1. — The taper reamer, *B*, Fig. 4, is for standard taper pins and has a taper of $\frac{1}{4}$ inch per foot. The diameter at the large end of the flutes is to be 0.720 inch. The diameter at the point of the reamer must be 0.580 inch, in order to accommodate the longest taper pins of this size made. How long should the fluted part of the reamer be made?

The taper per foot is $\frac{1}{4}$, or 0.250, inch, and the taper per inch equals 0.250 divided by 12, or 0.0208 inch. The taper in the length of reamer required is equal to the difference between the large and the small diameters, or 0.720 − 0.580 equals 0.140 inch. This amount of taper divided by the taper in one inch gives the required length of the flutes; thus, 0.140 divided by 0.0208 equals 6.731, which represents the length of flutes required. This dimension is nearly $6\frac{3}{4}$ inches, and, being a length dimension of no particular importance, it would be made to an even fractional part of an inch.

Example 2. — At *C*, Fig. 4, is shown a taper master gage intended for inspecting taper ring gages of various dimensions. The smallest diameter of the smallest ring gage is $1\frac{3}{4}$ inch, and the largest diameter of the largest ring gage is $2\frac{3}{4}$ inches. The taper per foot is $1\frac{1}{2}$ inch. It is required that the master gage extend one inch outside of the gages at both the small and the large ends, when these are tested. How long should the gage portion of this piece of work be?

The taper per foot is $1\frac{1}{2}$ inch, which is equivalent to $\frac{1}{8}$ inch taper per inch. The total taper from x to y is $2\frac{3}{4}$ minus $1\frac{3}{4}$, or one inch. Therefore, as the taper per inch, $\frac{1}{8}$, is contained in the taper of one inch in the distance from x to y exactly 8 times, the dimension from x to y is 8 inches. The gage extends one inch beyond x and y, respectively, at either end, and the total length of the gage is, therefore, 10 inches.

Figuring Offset of Tailstock for Taper Turning. — When a lathe is not provided with a taper attachment, the tailstock center is set over from its central position an amount depending upon the amount of taper and length of the part to be turned. This offset adjustment may easily be calculated approximately. If the tail-center is moved out of alignment with the live-center an amount A, as shown in Fig. 5, then the center of the work at the tail-center end will come nearer to the line of traverse BC of the tool than the center of the work at the live-center end, and the diameter of the piece, when turned, will be smaller at the tail-center than at the live-center.

When the tail-center is set over an amount A, the radius at the small end will be a dimension D smaller than the radius at the large end. This dimension D is also equal to the amount A which the tail-center has been set over, and the taper of the work in the length between the centers, therefore, is two times the amount the tailstock is set over; or, in other words, the tailstock is set over one-half of the taper in the length of the work.

When Taper per Foot and Length are known. — The amount which the tailstock must be set over can be determined if the taper per foot of the work and the length are known. Assume that a piece of work, $7\frac{1}{2}$ inches long, is required to be turned with a taper per foot equal to $\frac{3}{4}$ inch. We must first know how much the work tapers in $7\frac{1}{2}$ inches. This is found by dividing $\frac{3}{4}$ by 12, and multiplying the quotient by $7\frac{1}{2}$:

$$\left(\frac{3}{4} \div 12\right) \times 7\frac{1}{2} = \frac{15}{32}.$$

The taper in $7\frac{1}{2}$ inches, thus, is $\frac{15}{32}$ inch, and as the tailstock is moved one-half of this, it is set over $\frac{15}{64}$ inch.

When the taper per foot and the length of the work are given, the amount to set over the tailstock can be calculated from the following formula:

$$\text{Amount to set over tailstock} = \frac{1}{2} \times \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right).$$

Expressed in words, this formula reads:

To find the amount to set over the tailstock when the taper per foot and the length of the work are known, divide the taper per foot by 12, multiply the quotient by the length of the work, and divide the result by 2. (To divide by 2 is the same as to multiply by $\frac{1}{2}$.)

Owing to the fact that the work is not supported by the lathe centers at its extreme ends, but that the lathe centers enter into the work and support it at points a short distance from the ends, it is not practicable to calculate the amount to set over the tailstock so definitely that the taper can be

Fig. 5. Diagram illustrating how Tailstock Center of Lathe may be Offset for Taper Turning

turned to exact dimensions without a trial cut; but the calculation for setting over the tailstock gives a close approximation, and when a trial cut on the work has been taken, the final adjustment of the tailstock to obtain the correct taper can be made easily.

When the Diameters at Both Ends of a Tapered Piece are known. — If the diameters at both the large and small ends of work tapering for its full length are given, the amount to set over the tailstock can be determined without knowing the taper per foot, because all that is necessary to know is the taper in the length between the centers of the lathe. If, for instance, the diameter at the large end of the work is $1\frac{1}{2}$ inch and the diameter at the small end, $1\frac{1}{4}$ inch, as shown at *A* in Fig. 6, the amount to set over the tailstock will be one-half of the difference between the large and small diameters, or $\frac{1}{8}$ inch.

To find the amount to set over the tailstock for work tapering for its full length, when the diameters at the large and small ends are known, subtract the small diameter from the large, and divide the remainder by 2.

When Work is Part Straight and Part Tapered. — If part of the work is turned straight and part of it turned tapered, as

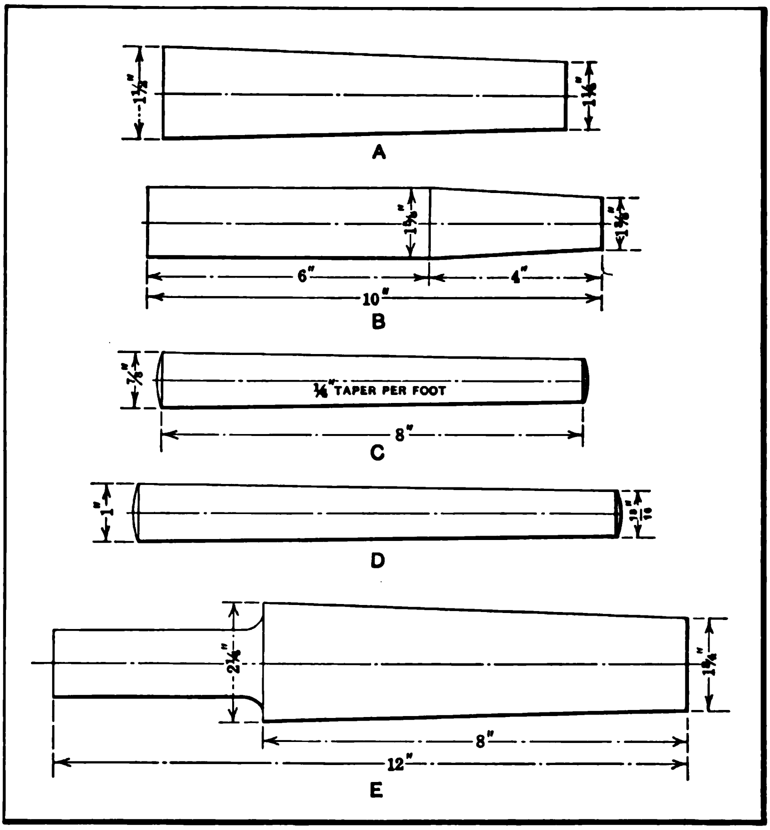


Fig. 6. Different Classes of Problems encountered in connection with Taper Turning

shown at *B*, Fig. 6, the taper in the whole length of the work must be determined, and then the tailstock set over one-half of this amount. The piece shown is $1\frac{3}{8}$ inch at the small end of the taper. It is tapered for 4 inches, and the diameter *at the large end* of the taper is $1\frac{5}{8}$ inch. It is then turned straight for the remaining 6 inches, the total length being 10 inches. We must first find what the taper would be in 10 inches if the whole piece had been tapered with the same taper as now required for 4 inches. The taper in 4 inches is $1\frac{5}{8} - 1\frac{3}{8} = \frac{1}{4}$ inch. The taper in 1 inch is $\frac{1}{16}$ inch, and in 10 inches, $10 \times \frac{1}{16} = \frac{5}{8}$ inch. The amount to set over the tailstock is one-half of this, or $\frac{5}{16}$ inch. If in this case the diameter at the small end were not given, but the taper per foot of the tapered part given instead, the taper in the total length of the work could be found directly; if the taper per foot be $\frac{3}{4}$ inch, the taper in 10 inches is $(\frac{3}{4} \div 12) \times 10 = \frac{5}{8}$ inch. Therefore, the amount to set over the tailstock is $\frac{5}{16}$ inch. The following formula is used when part of the work is turned straight and part tapered:

$$\text{Amount to set over tailstock} = \frac{1}{2} \times \left(\frac{\text{taper per foot}}{12} \times \text{total length of work}\right).$$

Expressed as a rule, this formula would read:

To find the amount to set over the tailstock for work partly tapered and partly straight, when the taper per foot and the total length of the work are known, divide the taper per foot by 12, multiply the quotient thus obtained by the total length of the work, and divide by 2.

If the taper per foot is not given, it must be found before using this formula and rule.

Examples for Practice. — The following examples will help to give a clear idea of the application of these rules.

Example 1. — The taper pin shown at *C*, Fig. 6, is 8 inches long, and tapers $\frac{1}{4}$ inch per foot. How much should the tailstock be set over when turning this pin?

Dividing the taper per foot by 12 gives 0.0208. Multiplying this figure (which represents the taper per inch) by 8 gives 0.166 as the taper in 8 inches. Dividing this by 2 gives the amount required to set over the tailstock. This amount is 0.083 inch.

Example 2. — Another taper pin, shown at *D*, Fig. 6, is 1 inch in diameter at the large end, and $\frac{13}{16}$ inch at the small end. How much should the tailstock be set over for turning this pin?

The total taper of this pin is found by subtracting the diameter at the small end, $\frac{13}{16}$ inch, from the diameter at the large end, 1 inch. This gives a remainder of $\frac{3}{16}$. One-half of this amount, or $\frac{3}{32}$ inch, represents the amount which the tailstock should be set over.

Example 3. — The diameter at the large end of the taper gage shown at *E*, Fig. 6, is $2\frac{1}{4}$ inches, the diameter at the small end is $1\frac{3}{4}$ inch, the length of the taper, 8 inches and the total length, 12 inches. How much should the tailstock be set over?

Subtracting the diameter at the small end, $1\frac{3}{4}$ inch, from the diameter at the large end, $2\frac{1}{4}$ inches, gives a taper of $\frac{1}{2}$ inch in 8 inches. Dividing $\frac{1}{2}$ by 8 gives the taper in one inch, which is $\frac{1}{16}$ inch. Multiplying this by the total length of the work, 12 inches, gives $\frac{3}{4}$ inch, which, divided by 2, gives finally, the required amount which the tailstock is to be set over. This latter is, therefore, set over $\frac{3}{8}$ inch.

CHAPTER VII

SPEEDS OF PULLEYS AND GEARING

THE relative speeds of different parts of a machine or of pulleys and gears which serve to transmit motion must be adapted to working requirements. If a grinding wheel is to be driven from a line of shafting, this wheel will not grind to the best advantage unless it runs at a certain speed, but if the speed is excessive, the wheel may be a source of danger and may even burst as the result of centrifugal force. In the case of a machine having different rotating parts, the speed of each shaft with its gearing or pulleys is determined in accordance with the work or purpose of each part which is in motion. In problems relating to speeds, the speed required for the driven part may be known, or the object may be to determine its speed when the driver rotates at a given speed and transmits the motion by means of pulleys of known diameter, or through gearing of a given size. The various classes of problems pertaining to simple and compound belt-and-pulley drives and different types and combinations of gearing will be explained in this chapter.

Speed of Driven Pulley required. — To find the number of revolutions per minute of the driven pulley when the diameter and the revolutions per minute of the driving pulley and the diameter of the driven pulley are known.

Rule: Multiply the diameter of the driving pulley by its number of revolutions per minute, and divide the product by the diameter of the driven pulley.

Example. — If the diameter of the driving pulley shown at *A*, Fig. 1, is 15 inches, and it makes 150 revolutions per minute, and the diameter of the driven pulley *B* is 9 inches, the number of revolutions per minute of *B* equals:

$$\frac{15 \times 150}{9} = 250 \text{ revolutions per minute.}$$

Diameter of Driven Pulley required. — To find the diameter of the driven pulley when the diameter and the number of revolutions per minute of the driving pulley and the number of revolutions per minute of the driven pulley are known.

Rule: Multiply the diameter of the driving pulley by its number of revolutions per minute, and divide the product by the number of revolutions per minute of the driven pulley.

Example. — If the diameter of the driving pulley *A*, Fig. 1, is 15 inches, and it makes 120 revolutions per minute, and the driven pulley *B* is required to make 200 revolutions per minute, the diameter of pulley *B* equals:

$$\frac{15 \times 120}{200} = 9 \text{ inches.}$$

Speed of Driving Pulley required. — To find the number of revolutions per minute of the driving pulley when the diameter and the number of revolutions per minute of the driven pulley and the diameter of the driving pulley are known.

Rule: Multiply the diameter of the driven pulley by its number of revolutions per minute, and divide the product by the diameter of the driving pulley.

Example. — If the diameter of the driven pulley *B*, Fig. 1, is 9 inches, and it makes 300 revolutions per minute, and the diameter of the driving pulley *A* is 15 inches, the number of revolutions per minute of pulley *A* equals:

$$\frac{9 \times 300}{15} = 180 \text{ revolutions per minute.}$$

Diameter of Driving Pulley required. — To find the diameter of the driving pulley when the diameter and the number of revolutions per minute of the driven pulley and the number of revolutions per minute of the driving pulley are known.

Rule: Multiply the diameter of the driven pulley by its number of revolutions per minute, and divide the product by the number of revolutions per minute of the driving pulley.

Example. — If the diameter of the driven pulley *B*, Fig. 1, is 9 inches, and it makes 205 revolutions per minute, and driving pulley *A* makes 123 revolutions per minute, the diameter of pulley *A* equals:

$$\frac{9 \times 205}{123} = 15 \text{ inches.}$$

Speed of Driven Pulley in Compound Drive. — When pulleys are arranged as shown in the lower part of Fig. 1,

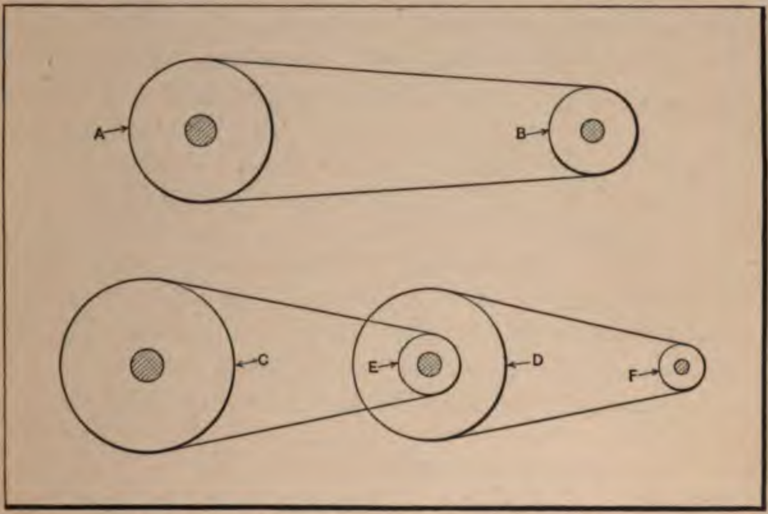


Fig. 1. Simple and Compound Belt and Pulley Drives

this is known as a compound drive. To find the number of revolutions per minute of the driven pulley when the diameters of all the pulleys and the number of revolutions per minute of the driving pulley are known.

Rule: Divide the product of the diameters of all the driving pulleys by the product of the diameters of all the driven pulleys, and multiply the quotient thus obtained by the number of revolutions per minute of the first driving pulley.

Example. — If, in the compound drive shown in Fig. 1, the diameter of the first driving pulley, *C*, is 18 inches, the diameter of the second driving pulley, *D*, is 16 inches, the diameter of the first driven pulley, *E*, is 6 inches, and the diameter of the last driven pulley, *F*, is 4 inches, and the first driving pulley *C* makes 120 revolutions per minute, the number of revolutions per minute of driven pulley *F* equals:

$$\frac{18 \times 16}{6 \times 4} \times 120 = 1440 \text{ revolutions per minute.}$$

Speed of Driving Pulley in Compound Drive. — To find the number of revolutions per minute of the driving pulley when the diameters of all the pulleys and the revolutions per minute of the last driven pulley are known.

Rule: Divide the product of the diameters of all the driven pulleys by the product of the diameters of all the driving pulleys, and multiply the quotient thus obtained by the number of revolutions per minute of the last driven pulley.

Example. — If, in the compound drive shown in Fig. 1, the diameter of the last driven pulley, *F*, is 4 inches, the diameter of the second driven pulley, *E*, 6 inches, the diameter of the first driving pulley, *C*, 18 inches, and the diameter of the second driving pulley, *D*, 16 inches, and the last driven pulley *F* makes 1440 revolutions per minute, the number of revolutions per minute of the first driving pulley *C* equals:

$$\frac{4 \times 6}{16 \times 18} \times 1440 = 120 \text{ revolutions per minute.}$$

To Find Diameters of Pulleys in Compound Drive. — To find the diameters of four pulleys *C*, *D*, *E*, and *F*, arranged as shown in Fig. 1, when the driving pulley *C* makes 120 revolutions per minute and the driven pulley *F* makes 1440 revolutions per minute.

Rule: Reduce to its lowest terms a fraction which has as its denominator the number of revolutions per minute of the driven pulley *F* and as its numerator the number of revolutions per minute of the driving pulley *C*. Now resolve the numerator thus found into two factors. Also resolve the denominator into two factors. Multiply one factor in the numerator and one factor in the denominator by some number which will give the diameters of one driven pulley and one driving pulley, respectively. Now multiply the remaining factor in the numerator and the remaining factor in the denominator by some number which will give the diameters of the other driven pulley and the other driving pulley, respectively.

Example. — If the driving pulley *C*, Fig. 1, makes 120 revolutions per minute, and the driven pulley *F* makes 1440 revolutions per minute, the diameters of four pulleys *C*, *D*, *E*, and *F* which will give the required speed ratio can be found as follows: First write the number of revolutions per minute of the driving pulley *C* as the numerator and the number of revolutions per minute of the driven pulley *F* as the denominator of a fraction, thus: $\frac{120}{1440}$, which reduced to its lowest terms equals $\frac{1}{12}$. This represents the required speed ratio between the driving and the driven pulley. Now resolve both the numerator and the denominator into two factors: $\frac{1}{12} = \frac{1 \times 1}{3 \times 4}$. Now multiply each pair of factors by trial numbers. If the numbers 6 and 4 are selected, then:

$$\frac{(1 \times 6) \times (1 \times 4)}{(3 \times 6) \times (4 \times 4)} = \frac{6 \times 4}{18 \times 16}.$$

The diameters of the driven pulleys *E* and *F* are equal to the two values 6 and 4 found in the numerator; the diameters of the driving pulleys *C* and *D* are equal to the values 18 and 16 found in the denominator. The pitch diameters of gears could be determined in the same way.

Influence of Belt Thickness on Pulley Speed. — When the diameters of the pulleys are small and the belt is relatively thick, the thickness of the belt should be taken into consideration in pulley speed calculations, especially when the difference of the two pulley diameters is great. In ordinary pulley calculations, however, where the pulleys are, say, 12 inches in diameter or more, it is not customary to consider the thickness of the belt. In the case of a feed belt of an engine lathe, the thickness of the belt makes an appreciable difference in the ratio of the pulley speeds. If the driving pulley is, say, 6 inches in diameter and the driven pulley is 2 inches in diameter, then the ratio of the driving and driven diameters, not considering the belt, would be as 6 to 2, or as 3 to 1. Considering the thickness of the belt, however, to be $\frac{1}{4}$ inch and taking this thickness into consideration, the ratio would be as $6\frac{1}{4}$ to $2\frac{1}{4}$, or as 2.77 to 1. It will be seen, therefore, that the thickness

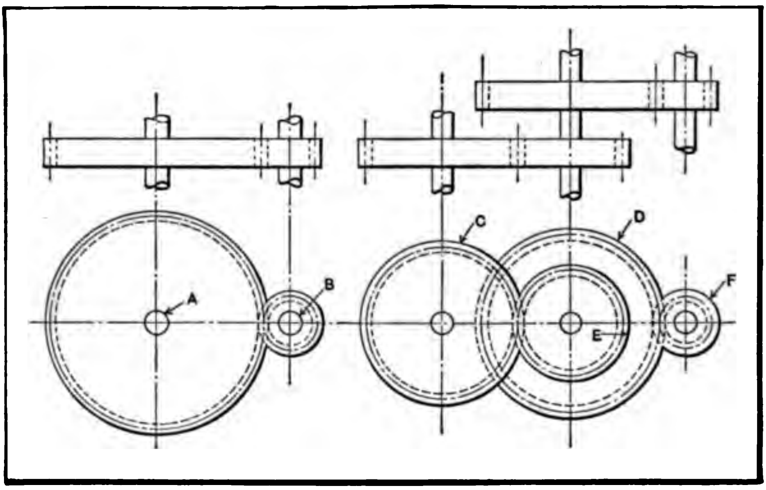


Fig. 2. Simple and Compound Gear Drives

of the belt makes a difference of about 8 per cent in the speed of the driven pulley.

Speeds of Gearing. — When gearing is to be employed to transmit motion and power from one shaft to another, it is often necessary or desirable that the ratio between the speeds of the driving and the driven shafts be made to equal some predetermined ratio. Also when gearing is already installed and in operation, it is frequently necessary to determine the exact ratio between the speeds of the driving and the driven shafts. The following rules and examples are applicable to such calculations.

Simple Spur Gearing. — A simple spur-gear drive consisting of a driving and a driven gear (such as shown at *A* and B, Fig. 2) will be considered first. Assume that the driving shaft B is required to make four revolutions while the driven shaft A makes one revolution. The ratio of the required gearing would, therefore, be 4 to 1 and the gear on shaft A would have four times as many teeth as the gear on shaft B. If the gear on shaft B has 12 teeth, the gear on shaft A must have 48 teeth, in order that all the teeth of the gear on shaft B may be engaged four times during one complete revolution of shaft A. If the relative speeds of shafts A and B are known, and also the number of teeth in one of the gears, the number of teeth required in the other gear can be found. Also, if the number of teeth in each of the gears is given and the number of revolutions per minute of one gear is known, the number of revolutions per minute of the other gear can be found. If only the speed ratio of the two gears is known, the number of teeth that is required in each of the two gears to produce the required ratio can be determined.

Speed of Driven Gear required. — When the speed of the driving gear and the number of teeth in the driving and the driven gears are known, the number of revolutions per minute of the driven gear can be found by the following rule:

Rule: Multiply the number of teeth in the driving gear by its number of revolutions per minute, and divide the product by the number of teeth in the driven gear.

Example. — If driving gear B, Fig. 2, has 12 teeth, and makes 260 revolutions per minute, and driven gear A has 48 teeth, the number of revolutions per minute of the driven gear A equals:

$$\frac{12 \times 260}{48} = 65 \text{ revolutions per minute.}$$

Number of Teeth in Both Gears required to obtain a Given Speed. — If the speed ratio between the driving and the driven shafts is known, the number of teeth required in each of two gears to produce the given ratio can be found by the following rule:

Rule: Write the speed ratio in the form of a fraction and multiply the numerator and the denominator by some number, thus obtaining a new fraction of the same value, which has a numerator equivalent to a suitable number of teeth for one gear and a denominator equivalent to a suitable number of teeth for the other gear.

Example. — If the speed ratio between the shafts *A* and *B*, Fig. 2, is $\frac{1}{4}$, or, as it is commonly expressed, 1 to 4, the number of teeth in each of two gears to give the required ratio can be found in the following manner: Write the ratio as a fraction, thus, $\frac{1}{4}$, and multiply both the numerator and the denominator by some trial number. As the numerator is 1 in this case, the trial number should be some number not less than 12, as gears having less than 12 teeth do not operate satisfactorily. Taking 14 as a trial number, we have $\frac{(1 \times 14)}{(4 \times 14)} = \frac{14}{56}$. The number of teeth in the gear having the greatest speed therefore, is 14, and the number of teeth in the other gear is 56.

Number of Teeth in Driven Gear required. — If the number of teeth in the driving gear and the number of revolutions per minute of both the driving and driven gears are known, the number of teeth in the driven gear can be found by the following rule:

Rule: Multiply the number of teeth in the driving gear by its number of revolutions per minute, and divide the product by the number of revolutions per minute of the driven gear.

Example. — If the driving gear *B* has 12 teeth and makes 144 revolutions per minute, and *A* makes 36 revolutions per minute, the number of teeth in gear *A* equals:

$$\frac{12 \times 144}{36} = 48 \text{ teeth.}$$

Pitch Diameter of Driven Gear required. — If the pitch diameters of the gears are substituted in place of the number of teeth in connection with speed calculations, the same results will be obtained. If driving gear *B* has a pitch diameter of 4 inches and it makes 144 revolutions per minute, and *A* makes 36 revolutions per minute, the pitch diameter of *A* equals:

$$\frac{4 \times 144}{36} = 16 \text{ inches.}$$

Number of Teeth in Driving Gear required. — If the number of revolutions per minute of the driving and the driven gears and the number of teeth in the driven gear are given, the number of teeth required in the driving gear can be found by the following rule:

Rule: Multiply the number of teeth in the driven gear by its number of revolutions per minute, and divide the product by the number of revolutions per minute of the driving gear.

Example. — If driven gear *B*, Fig. 2, has 12 teeth and makes 300 revolutions per minute, and driving gear *A* makes 75 revolutions per minute, the number of teeth in gear *A* equals:

$$\frac{12 \times 300}{75} = 48 \text{ teeth.}$$

Speeds and Sizes of Gears in Compound Gear Drive. — The gears *C*, *D*, *E*, and *F* shown in Fig. 2 form a compound gear drive. This method of employing gears to transmit motion and power from one shaft to another through intermediate gears keyed to an intermediate shaft makes it possible to obtain a relatively large reduction or increase in the speed between the driving and the driven shafts.

Speed of Driven Gear in Compound Gear Drive. — When the number of teeth in all of the gears is known and the number of revolutions per minute of the driving gear is given, the number of revolutions per minute of the driven gear can be found by the following rule:

Rule: Multiply the number of revolutions per minute of the driving gear by a fraction the numerator of which consists of the product of the number of teeth in each of the driving gears, and the denominator of which consists of the product of the number of teeth in each of the driven gears.

Example. — If the driving gear *F* (Fig. 2) makes 504 revolutions per minute and has 12 teeth, and second driving gear, *E*, has 24 teeth, first driven gear, D, 42 teeth, and second

Combination Belt and Gear Drive. — A combination of belt and gear drive is often employed in transmitting motion or power from one shaft to another. The calculations required in solving problems of this kind can be simplified if the gears are considered as pulleys having diameters equal to their pitch diameters. When this is done, the rules that apply to compound belt drives can be used in determining the speed or size of the gears or pulleys.

Example. — The following example illustrates the method of calculating the speed of a driven shaft in a combination belt and gear drive when the diameters of the pulleys and the

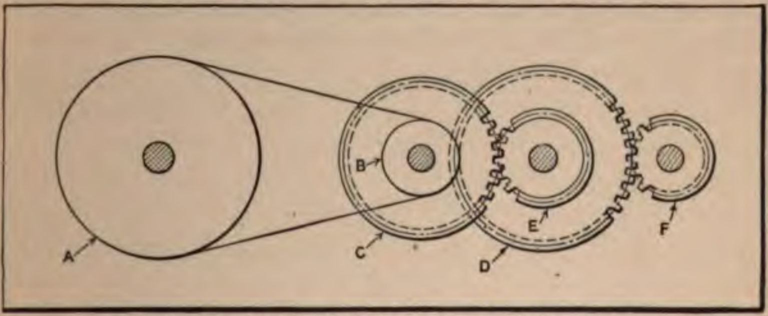


Fig. 3. Combination Pulley and Compound Gear Drive

pitch diameters of the gears are known, and the number of revolutions per minute of the driving shaft is given. If driving pulley *A*, Fig. 3, is 16 inches in diameter, and driven pulley *B*, 6 inches in diameter, and the pitch diameter of driving gear *C* is 12 inches, driving gear *D* is 14 inches, driven gear *E*, 7 inches, driven gear *F*, 6 inches, and driving pulley *A* makes 60 revolutions per minute, the number of revolutions per minute of *F* equals:

$$\frac{16 \times 12 \times 14}{6 \times 7 \times 6} \times 60 = 640 \text{ revolutions per minute.}$$

If the number of teeth in each gear is substituted for its pitch diameter, the result will be the same as when the pitch diameters are used.

Speeds of Bevel-gear Drives. — The rules for calculating the speeds and sizes of spur gears also apply to bevel gears.

Thus, in determining the speed of the driving or the driven shafts of a bevel-gear drive, or in calculating the size of gears needed to give any required speed ratio, the number of teeth in a bevel gear can be used the same as though it were a spur gear.

Example. — If bevel gear *A* (Fig. 4) has 20 teeth and makes 80 revolutions per minute and the bevel gear *B* has 40 teeth, the number of revolutions per minute of *B* equals:

$$\frac{20 \times 80}{40} = 40 \text{ revolutions per minute.}$$

The pitch diameters of bevel gears can also be used in calculating the speeds instead of the number of teeth.

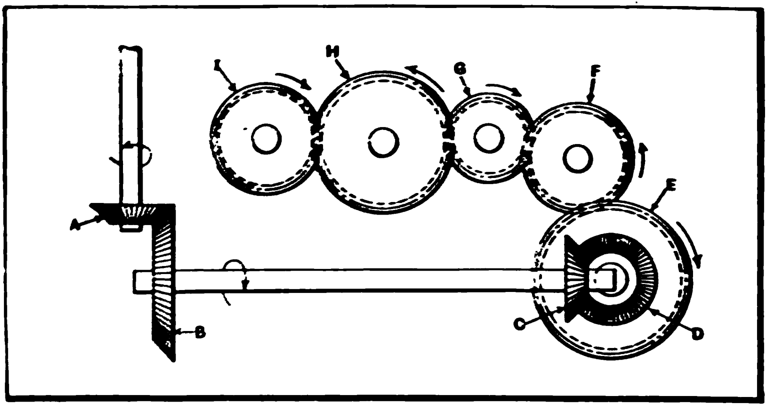


Fig. 4. Train of Bevel and Spur Gearing

Effect of Idler Gears. — When idler gears are used in spur-gear trains, the speed of the driven shaft is not affected by the idler gear or gears, but its direction of rotation is changed. If we assume that *E* (Fig. 4) is the driving gear and *I* the driven gear, the speed or size of either gears *E* or *I* can be calculated without taking into consideration the size of idler gears *F*, *G*, and *H*, as they have no effect whatever on the speed ratio between *E* and *I*.

Direction of Rotation. — The following rule can be used to determine the direction in which a driven shaft will be rotated when the driving gear transmits motion to the driven gear through one or more idler gears.

Rule: When one idler gear or any odd number of idler gears are interposed between the driving and the driven gear, the driven gear will be rotated in the same direction as the driving gear. When two or any even number of idler gears are interposed between the driving and the driven gears, the direction of rotation of the driven gear will be opposite to that of the driving gear. For example, it will be seen that driven gear *I* (Fig. 4) is rotated in the same direction as driving gear *E* and that there is an odd number of idler gears between the driving and driven gears.

Speeds of Worm-gear Drives. — The ratio between the speed of a worm and its driven worm-wheel depends upon the number of threads in the worm and the number of teeth in the worm-wheel. The number of threads in the worm in this case does not refer to the number of threads per inch, but to the number of single threads which form the worm thread, there being one thread if the worm is single-threaded, two, if double-threaded, three, if triple-threaded, etc.

Speed of Worm-wheel required. — If the number of threads on the worm and its number of revolutions per minute are given, and the number of teeth in the worm-wheel is known, the number of revolutions per minute of the worm-wheel can be found by the following rule:

Rule: Divide the product of the number of threads in the worm, multiplied by its number of revolutions per minute, by the number of teeth in the worm-wheel.

Example. — If worm *A* (Fig. 5) is double-threaded and makes 120 revolutions per minute, and worm-wheel *B* has 40 teeth, the number of revolutions per minute of worm-wheel *B* equals:

$$\frac{120 \times 2}{40} = 6 \text{ revolutions per minute.}$$

Number of Teeth in Worm-wheel for a Given Speed. — If the number of threads in the worm and its number of revolutiontions per minute are given, the number of teeth needed in the worm-wheel to give any required speed can be found by the following rule:

Rule: Multiply the number of threads in the worm by its number of revolutions per minute, and divide the product by the number of revolutions per minute of the worm-wheel.

Example. — If the worm is triple-threaded and makes 180 revolutions per minute, and the worm-wheel is required to make 5 revolutions per minute, the number of teeth in the worm-wheel equals:

$$\frac{3 \times 180}{5} = 108 \text{ teeth.}$$

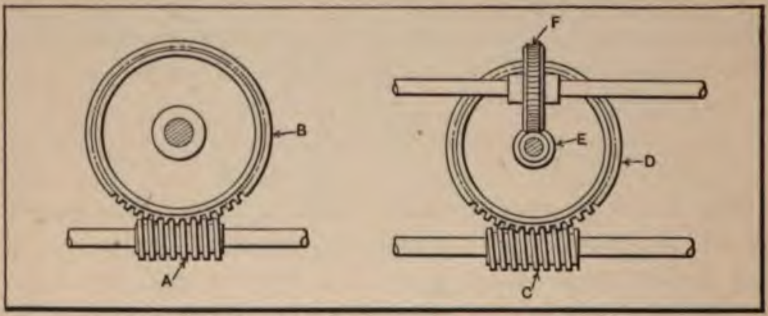


Fig. 5. Simple and Compound Worm-gear Drives

Speed of Driven Worm-wheel in Compound Drive. — The speed of the driven worm-wheel in a compound worm drive can be found by the following rule:

Rule: Multiply the number of revolutions per minute of the driving worm by a fraction the numerator of which consists of the product of the number of threads in all the worms, and the denominator of which consists of the product of the number of teeth in all the driven worm-wheels.

Example. — If worm C (Fig. 5) is single-threaded, worm E is double-threaded, worm-wheel D has 80 teeth, worm-wheel F, 40 teeth, and C makes 1600 revolutions per minute, the number of revolutions per minute of worm-wheel F equals:

$$1600 \times \frac{1 \times 2}{80 \times 40} = 1.$$

Speed of Driving Worm in Compound Worm-gear Drive. — If the number of threads in each of the driving worms, the number of teeth in each of the driven worm-wheels, and the number of revolutions per minute of the driven worm-wheel are known, the number of revolutions per minute of the driving worm can be found by the following rule:

Rule: Multiply the number of revolutions per minute of the driven worm-wheel by a fraction the numerator of which consists of the product of the number of teeth in the worm-wheels, and the denominator of which consists of the product of the number of threads in the worms.

Example. — If worm *C* (Fig. 5) is double-threaded, worm *E*, double-threaded, worm-wheel *D* has 80 teeth, worm-wheel *F*, 40 teeth, and *F* makes one turn per minute, the number of revolutions per minute of *C* equals:

$$\frac{40 \times 80}{2 \times 2} \times 1 = 800 \text{ revolutions per minute.}$$

Size of Worm-wheels and Number of Threads in Worms for Given Speed. — The number of teeth in the worm-wheels and the number of threads in each of the worms in a compound worm drive, to produce any required speed, may be found by the following rule:

Rule: Write the number of revolutions per minute of the driven worm-wheel as the numerator of a fraction and the number of revolutions per minute of the driving worm as the denominator. Reduce the fraction to its lowest terms. Resolve both the numerator and denominator into two factors. The two factors in the numerator then represent the number of threads required in the two driving worms and the two factors in the denominator represent the number of teeth required in the two driven worm-wheels.

Example. — If worm-wheel *F* (Fig. 5) makes 2 revolutions per minute and worm *C* makes 3200 revolutions per minute, the number of threads in the worms and the number of teeth in the worm-wheels can be found as follows: Write the number of revolutions made by the worm-wheel *F* as the numerator of a fraction and the number of revolutions made by the worm C as the denominator, thus, $\frac{2}{3200} = \frac{1}{1600}$; resolving the numerator and denominator into two factors, $\frac{1}{1600} = \frac{1 \times 1}{40 \times 40}$. Therefore, C will have one thread and E one thread, as indicated by the two factors in the numerator of the fraction. Also the worm-wheel D will have 40 teeth and the worm-wheel F, 40 teeth, as indicated by the two factors in the denominator.

Combination of Spur, Bevel, and Worm Gearing. — When a combination of spur, bevel, and worm gearing is employed to transmit motion and power from one shaft to another, the speed of the driven shaft can be found by the following method: Consider the worm as a gear having one tooth if it is single-threaded and as a gear having two teeth if double-threaded, etc. When this is done, the speed of the driving shaft can be found by applying the rules for ordinary compound spur gearing. If the pitch diameters of the gears are used instead of the number of teeth in making calculations, the worm should be considered as a gear having a pitch diameter of 1 inch, if a single-threaded, and 2 inches if a double-threaded worm, etc.

Example. — If driving spur gear A (Fig. 6) makes 336 revolutions per minute and has 42 teeth, driven spur gear B, 21 teeth, driving bevel gear C, 33 teeth, driven bevel gear D, 24 teeth, driving worm E, one thread, and driven worm-wheel F, 42 teeth, the number of revolutions per minute of F equals:

$$\frac{42 \times 33 \times 1}{21 \times 24 \times 42} \times 336 = 22 \text{ revolutions per minute.}$$

Relation of Peripheral Speeds to Pulley Diameters. — Grinding-machine and grinding-wheel catalogues generally contain a convenient reference table of correct wheel-spindle speeds in revolutions per minute, for peripheral speeds of 4000, 5000, 6000, and 6500 feet per minute. These tables, however, are computed for full-size wheels and are not correct for wheels reduced by wear. To compensate for this wheel wear, many grinding machines, whether for cylindrical or wet-tool grinding, snagging, etc., are made with two steps on the pulley. Then a worn wheel may be speeded up to its initial peripheral speed by shifting the spindle belt to the smaller spindle-pulley step. To insert an auxiliary table of spindle speeds in a general wheel catalogue would be out of the question, as the diameters of the smaller spindle pulleys vary on different makes of grinding machines. It has been left to the operator to shift his belt from the larger spindle pulley to the smaller when he thinks the change should be

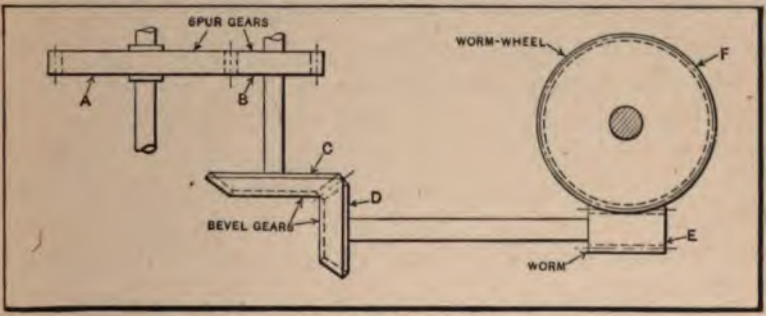


Fig. 6. Combination of Spur, Bevel, and Worm Gearing

made. The cutting action of a wheel often depends, to a great extent, upon its surface speed, and the time for making this change may be determined by means of a definite formula.

For instance, a 6-inch Norton plain grinding machine with a spindle speed of 1773 revolutions per minute, for a recommended peripheral speed of 6500 feet (as figured for a full-size 14-inch wheel for this size of machine), has two steps on the spindle pulley; the large step is 5.5 inches in diameter and the small step, 4 inches. What should be the minimum diameter of the wheel before the belt is shifted to the smaller step in order to obtain again a peripheral wheel speed of 6500 feet?

As the spindle makes 1773 revolutions per minute when the belt is on the large pulley, its speed with the belt on the smaller is $5.5 : 4 = x : 1773$, or $\frac{5.5 \times 1773}{4} = 2438$ revolutions per minute, approximately. To obtain the same peripheral speed as when the belt is on the large pulley, the diameters of the grinding wheel should be $14 : x = 2438 : 1773$, or $\frac{14 \times 1773}{2438}$ $= 10.18$ inches. Therefore, when the grinding wheel has been worn down to a diameter of 10.18 inches, or approximately $10\frac{3}{16}$ inches, the spindle belt should be shifted to the smaller step of the spindle pulley to obtain a peripheral speed of 6500 feet per minute. The method used in this example may be reduced to a formula for use with any make of grinding machine having a two-step spindle pulley.

Let D = diameter of wheel, full size;
D' = diameter of wheel, reduced size;
d = diameter of large pulley step;
d' = diameter of small pulley step;
V = revolutions per minute of spindle, using large pulley step;
v = revolutions per minute of spindle, using small pulley step.

Then
$$\frac{dV}{d'} = v; \text{ and } \frac{DV}{v} = D'.$$

CHAPTER VIII

CALCULATING CUTTING SPEEDS, FEEDS, AND MACHINING TIME

In the operation of various classes of machine tools such as are used for turning, planing, drilling, and milling castings and forgings, it is very essential to run the machine at the proper speed and to give the tool a feeding movement which is suitable for the work being operated upon. The selection of the proper cutting speed or rate of feed is based upon the different conditions governing each operation, and the machinist must be guided either by experience or by records of past performances. Sometimes it may be desirable to determine what cutting speed, in feet per minute, will be obtained for a given number of revolutions per minute; or this problem may be reversed, the object being to determine the speed of rotation required for a certain cutting speed. Problems also arise in connection with the rate at which a tool feeds while cutting. Typical speed and feed problems will be found in this chapter.

The meaning of the term "cutting speed" will first be explained. The cutting speed of a tool is the distance in feet which the tool point cuts in one minute; thus, if the point of a lathe tool cuts 40 feet, measured around the work, on the surface of a casting turned in the lathe, in one minute, the cutting speed is said to be 40 feet per minute.

On the planer, the cutting speed is equal to the length of cut that would be taken in one minute. If a cut 12 feet long is taken in 20 seconds, then, as 20 seconds is one-third of a minute, a cut 36 feet long could be made with the same speed in one minute, and the cutting speed is 36 feet per minute. The actual or net cutting speed, however, is reduced by the idle return stroke, as explained later.

When drilling a hole in the drill press, the cutting speed is the number of feet that the outer corners of the cutting edges travel in one minute.

Speed of Work for Given Diameter and Cutting Speed. — The problems in regard to cutting speeds in the lathe or turning and boring mill may be divided into two groups. The first problems to be considered are for determining the speed of the work in revolutions per minute when the diameter of the work turned in a lathe or boring mill and the required cutting speed are known.

Assume that the diameter *D*, Fig. 1, of the work is 5 inches, and the required cutting speed, 40 feet per minute. When the

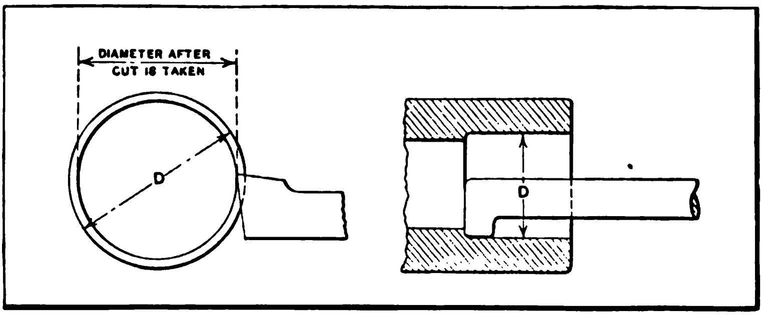


Fig. 1. Turning Tool and Work **Fig. 2. Boring Tool and Work**

diameter of the work is known, its circumference equals the diameter times 3.1416. Therefore, the circumference of the work in this case is $5 \times 3.1416 = 15.708$ inches. For calculations of this kind, it will be near enough to say that the circumference is 15.7 inches. For each revolution of the work, the length of its circumference passes the tool point once; thus for each revolution a length of 15.7 inches passes the tool. As the cutting speed is expressed in feet, the length 15.7 inches should also be expressed in feet, which is done by dividing by 12, thus obtaining $15.7 \div 12 = 1.308$ foot, as the circumference of the work. The next question is, how many revolutions, each equivalent to 1.308 foot, does it require to obtain a cutting speed of 40 feet? This is obtained by finding how many times 1.308 is contained in 40, or, in other words, by dividing 40 by 1.308. The quotient of this division is 30.6. Therefore, 30.6 revolutions per minute are required to obtain a cutting speed of 40 feet per minute in this case.

This calculation is expressed by the formula:

$$\text{Revolutions per minute} = \frac{\text{cutting speed in feet per minute}}{\left(\begin{matrix}\text{diameter of}\\ \text{work in inches}\end{matrix} \times 3.1416\right) \div 12}. \qquad (1)$$

If N = number of revolutions per minute, C = cutting speed in feet per minute, and D = diameter of work in inches, this formula can be written:

$$N = \frac{C}{(D \times 3.1416) \div 12}. \qquad (2)$$

If instead of turning work 5 inches in diameter, a hole 5 inches in diameter is to be bored by an ordinary forged boring tool (see Fig. 2) or a tool inserted into a boring bar, and the cutting speed is required to be 40 feet per minute, the calculation for the revolutions per minute is carried out in the same manner as above, and the same formulas are used, except that in the formula we write "diameter of hole to be bored in inches" instead of "diameter of work in inches."

For work done in the drill press, the formula can also be used by substituting "diameter of hole to be drilled in inches" for "diameter of work in inches."

The above case is a good example of the use of formulas in which letters are used for expressing a rule. If D = the diameter of work to be turned or the diameter D, Fig. 3, of the hole to be drilled or bored in inches, Formula (2) applies to both turned and bored or drilled work.

Cutting Speed for Given Diameter and Speed of Work. — When the number of revolutions which the work makes in a lathe or boring mill and the diameter are known, the cutting speed may be determined as illustrated by the following example:

A brass rod one inch in diameter is being turned. By counting the number of revolutions of the spindle of the lathe by means of a speed indicator, it is found that the work revolves 382 revolutions per minute. To find the cutting speed, the circumference of the work is first figured and changed into feet. The circumference in inches is $1 \times 3.1416 = 3.1416$, and $3.1416 \div 12 = 0.262$, the circumference in feet, or the distance passed over by the tool point for each revolution. During 382 revolutions, the distance passed over is $382 \times 0.262 = 100$ feet, which is thus the cutting speed per minute.

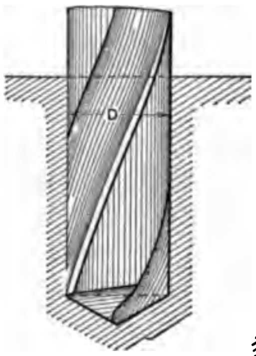


Fig. 3. Twist Drill

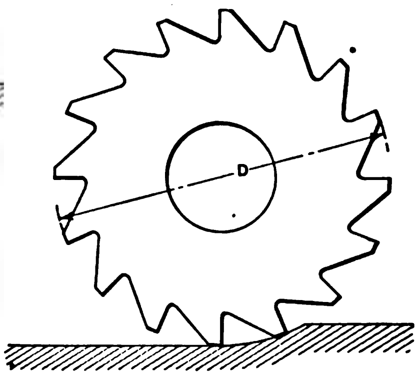


Fig. 4. Milling Cutter

This calculation is expressed by the formula:

$$\text{Cutting speed in feet per minute} = \frac{\text{diam. of work in inches} \times 3.1416}{12} \times \text{revolutions per minute.}$$

Using the same letters to denote the quantities in this formula as before, the formula may be written:

$$C = \frac{D \times 3.1416}{12} \times N. \qquad (3)$$

If, in this formula, D = diameter of work or diameter of bored or drilled hole in inches, the formula can be used for cutting speeds of drills and boring tools also.

(If the cut taken on a piece being turned is deep in proportion to the diameter of the work, it is preferable, in calculations for the cutting speed and revolutions per minute, to consider the *mean* diameter of the cut instead of the outside diameter of the work, and use the value for the mean diameter in the rules and formulas given. When the outside diameter and the depth of the cut are known, the mean diameter equals the outside diameter minus the depth of cut.)

Cutting Speeds of Milling Cutters. — The cutting speeds of milling cutters can be calculated when the diameter *D*, Fig. 4, of the cutter and the revolutions per minute are given. For instance, the diameter of a cutter is 6 inches and it makes 40 revolutions per minute. To find the cutting speed in feet per minute, first find the circumference of the cutter; thus, $6 \times 3.1416 = 18.8496$, or about 18.8 inches; change this to feet, thus, $18.8 \div 12 = 1.566$ feet. As the cutter makes 40 revolutions per minute, the cutting speed is 40 times the circumference, or $40 \times 1.566 = 62.64$ feet per minute.

If, in Formula (3), D = diameter of cutter, this formula can be used for finding the cutting speed of milling cutters.

If the required cutting speed of a cutter is given and its diameter known, and the number of revolutions at which it should be run are to be found, Formula (2) can be used, in this case D being the diameter of cutter.

Formulas and Rules for Calculating Cutting Speeds. — The following formulas and rules for calculating cutting speeds are a summary of those previously given.

If N = number of revolutions per minute, C = cutting speed in feet per minute, and D = the diameter of the work to be turned, the hole to be bored or drilled, or the diameter of the milling cutter, the following formulas are used:

$$N = \frac{C}{(D \times 3.1416) \div 12}; \qquad C = \frac{D \times 3.1416}{12} \times N.$$

These formulas may be expressed as rules as follows:

Rule 1: To find the number of revolutions per minute when the diameter of the work turned, the hole drilled or bored, or the milling cutter used, in inches, and the cutting speed in feet per minute are given, multiply the diameter by

3.1416 and divide the result by 12. Then divide the given cutting speed by the quotient thus obtained.

Rule 2: To find the cutting speed in feet per minute when the diameter of the work to be turned, the hole drilled or bored, or the milling cutter used is given in inches, and the number of revolutions per minute are known, multiply the diameter by 3.1416 and divide the result by 12. Then multiply the quotient thus obtained by the number of revolutions per minute.

Feed of Cutting Tools. — The feed of a lathe tool is its sidewise motion (traverse) for each revolution of the work· thus, if the feed is $\frac{1}{32}$ inch, it means that for each revolution of the work the lathe carriage and tool move $\frac{1}{32}$ inch along the lathe bed, thus cutting a chip $\frac{1}{32}$ inch wide.

The feed of a drill in the drill press is the downward motion of the drill per revolution. The feed of a milling cutter is the forward movement of the milling machine table for each revolution of the cutter.

Sometimes the feed is expressed as the distance which the drill or the milling machine table move forward in one minute. In order to avoid confusion, it is, therefore, always best to state plainly in each case whether feed per revolution or feed per minute is meant.

Time required for Turning Work in the Lathe. — The most common calculation in which the feed of a lathe tool enters is the time required for turning or boring a given piece of work, when the feed, cutting speed, and the diameter of work (or the number of revolutions per minute) are known.

Example. — Assume that a tool-steel arbor, 2 inches in diameter, is to be turned. The length to be turned on the arbor (the length of cut) is 10 inches. The cutting speed is 25 feet per minute and the feed or traverse of the cutting tool is $\frac{1}{32}$ inch per revolution. How long a time would it require to take one cut over the surface of the work?

First find the number of revolutions per minute of the work:

$$\frac{25}{(2 \times 3.1416) \div 12} = \frac{25}{0.524} = 47.7.$$

As the tool feeds forward $\frac{1}{32}$ inch for each revolution of the work, it is fed forward $47.7 \times \frac{1}{32}$, or 1.49 inch in one minute. The time required to traverse the whole length of the work, 10 inches, is obtained by finding how many times 1.49 is contained in 10, or by dividing 10 by 1.49. The quotient of this division is 6.71 minutes. It would thus take $6\frac{3}{4}$ minutes, approximately, for the tool to traverse the work once with the cutting speed and feed given.

Expressed in a formula, the calculation takes this form:

$$\text{Time required for one cut over the work} = \frac{\text{length of cut}}{\text{rev. per min.} \times \text{feed per revolution}}.$$

If T = time required for one complete cut over the work, in minutes;
L = length of cut, in inches;
N = revolutions per minute;
F = feed per revolution, in inches;

then the formula above can be written:

$$T = \frac{L}{N \times F}.$$

Expressed as a rule, the formula would be:

Rule: To find the time required to take one complete cut over a piece of work in the lathe when the feed per revolution, the total length of cut, and the number of revolutions per minute are given, divide the total length of the cut by the number of revolutions per minute multiplied by the feed per revolution.

If the cutting speed and diameter of work are given instead of the number of revolutions, first find the revolutions before applying the formulas or rules above. When the feed per revolution is known, the feed per minute equals the revolutions per minute times the feed per revolution.

Time required for Drilling. — In order to calculate the time required for drilling a given depth of hole, the number of revolutions per minute of the drill, and the feed per revolution (or the cutting speed, the diameter of the drill and the feed per revolution) must be known.

Example. — Assume that a $1\frac{7}{8}$-inch drill makes 80 revolutions per minute and that the feed per revolution is 0.008 inch. How long a time would be required to drill a hole $5\frac{1}{2}$ inches deep?

To find the number of revolutions required to drill the full depth of the hole, divide $5\frac{1}{2}$ by 0.008, obtaining the quotient 687.5, or, approximately, 690 revolutions. As the drill makes 80 revolutions in one minute, find the total number of minutes required by dividing 690 by 80, the quotient 8.6 being the number of minutes required to drill a hole $5\frac{1}{2}$ inches deep under the given conditions. If, in the foregoing,

T = time required for drilling, in minutes,
L = depth of drilled hole, in inches,
N = number of revolutions per minute of the drill,
F = feed per revolution, in inches

then:

$$T = \frac{L}{N \times F}.$$

It will be seen that this formula is of the same form as the one for finding the time for turning or boring work in the lathe.

If the cutting speed of the drill and its diameter be given instead of the number of revolutions, find the number of revolutions before applying the formula above. If the feed per minute be given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

Time required for Milling. — The time required for milling may be found if the number of revolutions per minute of the cutter, and the feed per revolution (or the cutting speed, the diameter of the cutter and the feed per revolution) are known. If the feed per minute is given, the feed per revolution can be found by dividing the feed per minute by the number of revolutions per minute.

Example. — If the length of the cut taken in a milling machine is $8\frac{3}{8}$ inches and the feed is $\frac{1}{64}$ inch per revolution, how long a time will it take for a cutter making 20 revolutions per minute to traverse the work?

As the feed per revolution is $\frac{1}{64}$ inch and the cutter makes 20 revolutions per minute, the feed per minute is $\frac{20}{64}$, or $\frac{5}{16}$ inch. To find the time required for the cutter to traverse the full length of the work, divide the length of the cut, $8\frac{3}{8}$ inches, by the feed in one minute; thus:

$$8\frac{3}{8} \div \frac{5}{16} = \frac{67}{8} \times \frac{16}{5} = \frac{134}{5} = 26\frac{4}{5} = 26.8.$$

The time required would thus be 27 minutes, approximately.

If T = time required for the cutter to traverse the work, in minutes,
L = length of cut, in inches,
N = revolutions per minute of the cutter,
F = feed per revolution, in inches,

then:
$$T = \frac{L}{N \times F}.$$

It will be seen that the form of this formula is the same as that of the formulas for the time required for drilling and turning.

If the cutting speed and the diameter of the cutter are given instead of the number of revolutions, the latter number is first found before the formula above is applied.

To Calculate the Time required for Planing. — The time required for planing a piece of work can be calculated if the feed per stroke, and the number of cutting strokes of the planer table per minute, and the width of the work are known.

The feed of a planer tool is its sidewise motion for each cutting stroke of the table or platen. If for each cutting stroke the tool-carrying head moves $\frac{1}{16}$ inch along the cross-rail, we say that the feed is $\frac{1}{16}$ inch. Each cutting stroke necessitates a return stroke, and in the following, when the expression "number of strokes" is used, it means the number of cutting strokes.

Example. — Assume that a planer makes 6 cutting strokes per minute, that the feed per stroke is $\frac{3}{32}$ inch, and that the width of the work is 22 inches. Find the time required for planing the work.

As the planer makes 6 strokes per minute and the feed per stroke is $\frac{3}{32}$ inch, the feed per minute is $6 \times \frac{3}{32}$, or $\frac{9}{16}$ inch. The tool must traverse 22 inches to plane the complete work; the traverse in one minute being $\frac{9}{16}$ inch, the total number of minutes required to traverse the work is found by dividing 22 by $\frac{9}{16}$.

$$22 \div \frac{9}{16} = \frac{22}{1} \times \frac{16}{9} = \frac{352}{9} = 39\frac{1}{9} \text{ minutes.}$$

The time required for planing the work is thus 40 minutes, approximately.

This calculation may be summed up in the following formula, applicable to any case where the feed per stroke, the number of strokes per minute, and the width of the work are known:

$$T = \frac{W}{F \times N}.$$

In this formula,

T = time required for planing, in minutes;
W = width of work, in inches;
F = feed per stroke, in inches;
N = number of cutting strokes per minute.

The formula expressed as a rule would be as follows:

Rule: To find the time required for planing when the width of the work, the feed per stroke, and the number of cutting strokes per minute are known, divide the width of the work by the feed times the number of cutting strokes per minute.

To Calculate Cutting Speed and Return Speed. — The speed at which the platen returns when the cutting stroke is completed is usually two or more times the cutting speed. If the return speed is twice as fast as the cutting speed, we say that the *ratio* of return speed to cutting speed is 2 to 1. If the return speed is three times as fast as the cutting speed, we say that the ratio between the speeds is 3 to 1, and so on. Ordinarily, these ratios are designated "2," "3," etc. If the *return speed* is 100 feet and the cutting speed 50 feet per minute, the ratio is 2; while, if the return speed is 90 feet per minute and the cutting speed 30 feet, the ratio is 3.

If the number of cutting strokes per minute, the length of the stroke, and the ratio between cutting and return speeds are known, the cutting speed and return speed can be calculated. The number of strokes per minute can be counted and the length of the stroke can be measured in each case; the ratio between the return speed and the cutting speed is determined by the design of the planer, and for long strokes can be determined by taking the time required for the forward stroke and the return stroke by a watch having a second-hand or by a stop-watch. Thus the cutting speed and the return speed in feet per minute of any planer can be easily determined.

If C = cutting speed in feet per minute;
R = return speed of planer, in feet per minute:
N = number of cutting strokes per minute;
S = length of cutting stroke, in feet;
P = ratio between return speed and cutting speed;

then the following formulas for finding the cutting and return speeds can be used:

$$C = \frac{R}{P} = \frac{N \times S \times (P + 1)}{P}.$$

In this formula, the time lost when reversing is not considered.

$$R = N \times S \times (P + 1).$$

These formulas may be expressed in rules as follows:

Rule 1: To find the cutting speed in feet per minute when the return speed and the ratio of return speed to cutting speed are known, divide the return speed by the ratio.

Rule 2: To find the return speed in feet per minute when the number of strokes per minute, the length of the cutting stroke in feet, and the ratio of return speed to cutting speed are known, multiply the number of revolutions by the length of stroke, and multiply the product obtained by 1 added to the ratio.

To Find the Number of Strokes per Minute from the Cutting and Return Speeds. — If the cutting speed and the return speed of a planer are known, the number of cutting strokes per minute may be found if the length of the stroke is also known.

Example. — Assume that the cutting speed of a planer is 50 feet per minute, that the ratio between the return speed and the cutting speed is 2, and that the length of the stroke for planing a given piece of work is 10 feet. How many strokes per minute will the planer make?

As the ratio between the return speed and the cutting speed is 2, the return speed is twice the cutting speed, or 100 feet per minute. As the length of the stroke is 10 feet and the cutting speed is 50 feet per minute, the time required for the forward stroke is $\frac{1}{5}$ minute. The return speed being 100 feet per minute, $\frac{1}{10}$ of a minute will be required for the return stroke of 10 feet. The time required for a complete forward and return stroke, therefore, is:

$$\frac{1}{5} + \frac{1}{10} = \frac{2}{10} + \frac{1}{10} = \frac{3}{10} = 0.3 \text{ minute.}$$

The number of cutting strokes per minute is obtained by finding how many times 0.3 minute is contained in 1 minute, or by dividing 1 by 0.3. The quotient, $3\frac{1}{3}$, is the number of strokes per minute. The time lost at the moment of reversal has not been considered.

The foregoing calculation may be summed up in the following formula, applicable to any case where the cutting speed and the return speed and the length of the stroke are known:

$$N = \frac{1}{\frac{S}{C} + \frac{S}{R}}.$$

In this formula,

N = number of cutting strokes per minute;
S = length of stroke, in feet;
C = cutting speed, in feet per minute;
R = speed of return stroke, in feet per minute.

The formula expressed as a rule would be as follows:

Rule: To find the number of strokes per minute of a planer when the cutting and return speed and the length of the stroke are known, divide the length of the stroke by the cutting speed; then divide the length of the stroke by the return speed; add the two quotients, and divide 1 by the sum thus obtained.

Figuring the Net Cutting Speed of a Planer. — When considering the cutting speeds of a planer, it is well to remember that the speed of the table during the forward or cutting stroke is greater than the *net* cutting speed. For instance, if the speed during the forward or cutting stroke is 30 feet per minute and the table has a return speed of 90 feet per minute, the actual number of feet per minute traversed by the tool while cutting would be $22\frac{1}{2}$ feet per minute, the net speed being reduced because of the idle return period when the tool is not at work.

Considering the speeds just given, if the forward movement were equal to a length of 30 feet, one minute would be required, and since the return speed is three times as fast, the return stroke would be completed in $\frac{1}{3}$ minute; therefore, the total time for the forward and return strokes would be $1\frac{1}{3}$ minute. To obtain the net cutting speed, divide the speed during the forward or cutting stroke in feet per minute by the total time required for the forward and return strokes, assuming that the length of stroke were equal to the forward cutting speed. In this instance, the forward cutting speed is 30 and the total time required for the forward and return strokes is $1\frac{1}{3}$ minute. Hence, the net cutting speed equals:

$$30 \div 1\frac{1}{3} = \frac{30}{1} \div \frac{4}{3} = \frac{30}{1} \times \frac{3}{4} = 22.5 \text{ feet per minute.}$$

When the net cutting speed is known, the number of cutting strokes per minute may be determined by simply dividing the net cutting speed by the length of the stroke.

CHAPTER IX

CHANGE-GEARING FOR THREAD CUTTING AND SPIRAL MILLING

WHILE lathe operators ordinarily are not required to calculate the combinations of gearing to use for cutting screw threads of different pitch, the method of determining the right combination to use should be understood. The number of times that the spindle will revolve while the carriage moves one inch along the lathe bed is determined by the ratio of the change-gears. By employing different ratios of change-gearing, therefore, different numbers of threads per inch can be cut.

The change-gearing may be either *simple* or *compound*. Simple gearing is shown at *A*, Fig. 1. When simple gearing is used it is always necessary to use an idler between the gear on the spindle stud and the gear on the lead-screw. This idler has no influence on the ratio of the gearing, and can have any number of teeth. Compound change-gearing is shown at *B*.

Finding the Lathe Screw Constant. — In order to be able to calculate change-gears for the lathe, it is necessary first to find the "lathe screw constant." This constant is always the same for each particular lathe, but it may be different for lathes of different sizes or makes.

Rule: To find the screw constant of a lathe, place gears with an equal number of teeth on the spindle stud and the lead-screw. Then cut a thread on a piece of work in the lathe. The number of threads per inch that will be cut on the work when gears with equal numbers of teeth are placed as directed is called the "screw constant" of the lathe.

For example, put gears with 48 teeth on the spindle stud and on the lead-screw, and any convenient gear on the intermediate stud. Then cut a thread on a piece between the centers. If the number of threads per inch is found to be 8, the screw constant of this lathe is said to be 8.

Change-gears when Simple Gearing is used. — When the lathe screw constant has been found, the number of teeth to be used in the change-gears for cutting any number of threads within the capacity of the lathe can be determined as follows:

Rule: Place the lathe screw constant as the numerator and the number of threads per inch to be cut as the denominator of a fraction, and multiply the numerator and the

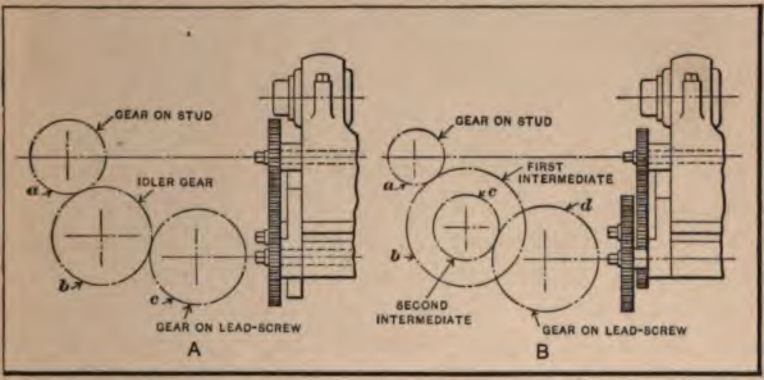


Fig. 1. Simple and Compound Gearing for Screw Cutting Lathe

denominator by the same number until a new fraction results where the numerator and denominator represent suitable numbers of teeth for the change-gears.

In the new fraction, the numerator gives the number of teeth in the gear on the spindle stud, and the denominator the number of teeth in the gear on the lead-screw. This rule can be more easily remembered if written as a formula:

$$\frac{\text{Lathe screw constant}}{\text{Threads per inch to be cut}} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead-screw}}.$$

Assume that 10 threads per inch are to be cut in a lathe where the lathe screw constant has been found to be 6. Also assume that the numbers of teeth in the available change-gears of this lathe are 24, 28, 32, 36, 40, etc., increasing by

4 up to 100. By substituting the figures given, in the formula above, and carrying out the calculation:

$$\frac{6}{10} = \frac{6 \times 4}{10 \times 4} = \frac{24}{40}.$$

By multiplying both the numerator and the denominator by 4, two available gears with 24 and 40 teeth, respectively, are obtained. The 24-tooth gear goes on the spindle stud, and the 40-tooth gear, on the lead-screw. It will be seen that if 6 and 10 had been multiplied by 5, the result would have been 30 and 50 teeth, which gears are not available in the set of gears with this lathe.

Assume that it is required to cut $11\frac{1}{2}$ threads per inch in the same lathe having the same set of change-gears. Then,

$$\frac{6}{11\frac{1}{2}} = \frac{6 \times 8}{11\frac{1}{2} \times 8} = \frac{48}{92}.$$

It will be found that multiplying by any other number than 8 would not, in this case, give numbers of teeth that could be found in the gears with the lathe. The lathe screw constant differs for different makes and sizes of lathes, and should be determined for each particular lathe.

Compound Gearing. — Sometimes it is not possible to obtain gears that will give the required ratio for the thread to be cut in a simple train, and then compound gearing must be employed. The method for finding the number of teeth in the gears in compound gearing is exactly the same as for simple gearing, except that we divide both the numerator and the denominator of the fraction, giving the ratio of screw constant to threads per inch to be cut, into two factors, and then multiply each "pair" of factors by the same number, in order to obtain the change-gears. (One factor in the numerator and one in the denominator make one pair.)

Assume that the lathe screw constant is 6, that the numbers of teeth in the available gears are 30, 35, 40, 45, 50, 55, etc., increasing by 5 up to 100. Assume that it is required to cut 24 threads per inch. Then, $\frac{6}{24}$ = ratio

By dividing the numerator and the denominator of the ratio into two factors and multiplying each pair of factors by the same number, as shown below, we find the gears:

$$\frac{6}{24} = \frac{2 \times 3}{4 \times 6} = \frac{(2 \times 20) \times (3 \times 10)}{(4 \times 20) \times (6 \times 10)} = \frac{40 \times 30}{80 \times 60}.$$

The four numbers in the last fraction give the numbers of teeth in the gears which should be used. The gears in the numerator, with 40 and 30 teeth, are the driving gears, and those in the denominator, with 80 and 60 teeth, are the driven gears. The driving gears are the gears *a* and *c* (see Diagram *B*, Fig. 1) and the driven gears, *b* and *d*. Either of the driving gears may be placed on the spindle stud, and either of the driven, on the lead-screw.

Sometimes the lead of a thread is given as a fraction of an inch instead of stating the number of threads per inch. For instance, a thread may be required to be cut having $\frac{3}{8}$ inch lead. In this case, the expression "$\frac{3}{8}$ inch lead" should first be transformed to "number of threads per inch," after which we can proceed to find the change-gears as previously explained. The number of threads (the thread being single) equals:

$$\text{Number of threads per inch} = \frac{1}{\frac{3}{8}} = 1 \div \frac{3}{8} = \frac{8}{3} = 2\tfrac{2}{3}.$$

To find the change-gears to cut $2\frac{2}{3}$ threads per inch in a lathe having a screw constant of 8 and change-gears running from 24 to 100 teeth, increasing by 4, proceed as below:

$$\frac{8}{2\frac{2}{3}} = \frac{2 \times 4}{1 \times 2\frac{2}{3}} = \frac{(2 \times 36) \times (4 \times 24)}{(1 \times 36) \times (2\frac{2}{3} \times 24)} = \frac{72 \times 96}{36 \times 64}.$$

Number of Threads per Inch obtained with a Given Combination of Gears. — If the problem is to determine the number of threads per inch that will be obtained with a given combination of gearing, the following rule may be applied.

Rule: Multiply the lathe screw constant by the number of teeth in the driven gear (or by the product of the numbers of teeth in both driven gears in the case of compound gearing), and divide the product thus obtained by the number of teeth in the driving gear (or by the product of the numbers of teeth in the two driving gears of a compound train). The quotient equals the number of threads per inch obtained with that combination of gearing.

Example. — When the driving gears in a compound train have 40 and 30 teeth, respectively, and the driven gears 80 and 60 teeth, how many threads per inch will be cut on a lathe equipped with this gearing, if the lathe screw constant is 6?

$$\text{Threads per inch} = \frac{\text{constant} \times \text{No. of teeth in driven gears}}{\text{number of teeth in driving gears}}$$

Therefore, in this example,

$$\text{Threads per inch} = \frac{6 \times 80 \times 60}{40 \times 30} = 24.$$

Example. — When the driving gear or the "gear on the stud" has 48 teeth, and the driven gear or the "gear on the lead-screw," 92 teeth, how many threads per inch will be cut, if the lathe screw constant is 6?

$$\text{Threads per inch} = \frac{6 \times 92}{48} = 11\tfrac{1}{2}.$$

Change-gears for Cutting Metric Threads. — The metric system of length measurement is in use in practically all countries except in the United States, Great Britain, and the British colonies. The unit of length in the metric system is the meter, which equals nearly 39.37 inches (or practically $39\frac{3}{8}$ inches).

In medium and small machine design the unit employed is almost always the millimeter. One millimeter equals 0.03937 inch; one inch equals $\frac{1}{0.03937}$, or 25.4 millimeters, almost exactly.

When screws are made in accordance with the metric system it is not the usual practice to give the number of threads per millimeter or centimeter in the same way as the number of threads per inch is given in the English system. Instead, the lead of the thread in millimeters is given. A screw thread is said to have 2 millimeters lead, 3 millimeters lead, 4.5 millimeters lead, etc.

It often happens that screws and taps having threads according to the metric system are required. This thread can be cut on a lathe having an English lead-screw, provided change-gears with the required number of teeth are used.

The first step in finding the change-gears is to find how many threads per inch there are in the screw to be cut, when the lead is given in millimeters. Assume that a screw is required with 3 millimeters lead. How many threads per inch are there in this screw? As there are 25.4 millimeters in one inch, we can find how many threads there would be in one inch, if we find how many times 3 is contained in 25.4; in other words, divide 25.4 by 3. It is not necessary to carry out the division; simply write it as a fraction in the form $\frac{25.4}{3}$, which implies that 25.4 is to be divided by 3. This fraction now gives the number of threads per inch to be cut. When this fraction has been obtained, proceed as if change-gears were to be found for cutting threads with English pitches. Place the lathe screw constant in the numerator of a fraction and the number of threads per inch to be cut in the denominator. If the screw constant of a lathe is 6 and the number of threads to be cut, $\frac{25.4}{3}$, as previously found, the ratio of the change-gearing is:

$$\frac{6}{\frac{25.4}{3}} = \text{ratio.}$$

This may seem complicated, but remembering that the line between the numerator and the denominator in a fraction means that the numerator is to be divided by the denominator, then by carrying out this division:

$$6 \div \frac{25.4}{3} = 6 \times \frac{3}{25.4} = \frac{6 \times 3}{25.4}.$$

The fraction $\frac{6 \times 3}{25.4}$ is the ratio of the change-gearing required, and all that has to be done now is to multiply the numerator and the denominator of this fraction by the same number until suitable numbers of teeth for the change-gears are found. By trial it is found that the first whole number by which 25.4 can be multiplied so as to obtain a whole number as a result, is 5. Multiplying 25.4 by 5 gives 127. Thus there must be one gear with 127 teeth whenever a metric thread is cut by means of an English lead-screw. The other gear required in this case has 90 teeth, because $5 \times 6 \times 3 = 90$. The calculation would be carried out as shown below:

$$\frac{6 \times 3 \times 5}{25.4 \times 5} = \frac{18 \times 5}{127} = \frac{90}{127}.$$

What has just been said can be expressed as follows:

Rule: To find the change-gears for cutting metric pitches with an English lead-screw, place the lathe screw constant multiplied by the number of millimeters lead of the thread to be cut multiplied by 5, in the numerator of the fraction, and 127 as the denominator. The product of the numbers in the numerator give the number of teeth in the gear on the spindle stud, and 127 is the number of teeth in the gear on the lead-screw. Written as a formula, this rule would be:

$$\frac{\text{Lathe screw constant} \times \text{lead of thread to be cut, in millimeters} \times 5}{127} = \frac{\text{teeth in spindle stud gear}}{\text{teeth in lead-screw gear}}$$

As an example, assume that a screw with 2.5 millimeters lead is to be cut on a lathe having a screw constant of 8. By placing the given figures in the formula:

$$\frac{8 \times 2.5 \times 5}{127} = \frac{100 \ldots \text{spindle stud gear}}{127 \ldots \text{lead-screw gear}}$$

Continued Fractions applied to Change-gear Calculations. — Continued fractions are sometimes employed to obtain a fraction which is small and convenient to use and which has *very* nearly the same value as a larger and more cumbersome fraction. The practical use of continued fractions as applied to shop work is in calculating change-gears such as might be required for cutting a screw thread having an unusual pitch. A continued fraction may be defined as a fraction having unity, or 1, for its numerator, and for its denominator some number plus some fraction which also has 1 for its numerator and for its denominator some number plus a fraction, etc.

If both the numerator and the denominator of the fraction $\frac{453}{1908}$ are divided by its numerator, the fraction becomes $\frac{1}{4\frac{96}{453}}$. This process may be continued by dividing the numerator and the denominator of the fraction $\frac{96}{453}$, and the same process repeated for other fractions that might be obtained. Thus,

$$\frac{453}{1908} = \frac{1}{4\frac{96}{453}}; \qquad \frac{96}{453} = \frac{1}{4\frac{69}{96}}; \qquad \frac{69}{96} = \frac{1}{1\frac{27}{69}};$$

$$\frac{27}{69} = \frac{1}{2\frac{15}{27}}; \qquad \frac{15}{27} = \frac{1}{1\frac{12}{15}}; \qquad \frac{12}{15} = \frac{1}{1\frac{1}{4}}.$$

If the fractions obtained by dividing the numerators and the denominators are written down without the fractional part of the denominator, we have, in this case, $\frac{1}{4}, \frac{1}{4}, \frac{1}{1}, \frac{1}{2}, \frac{1}{1}, \frac{1}{1}, \frac{1}{4}$. As will be seen, the numerators are 1 in each case, and the denominators are quotients obtained by dividing the different denominators by their numerators. In this way, the continued fraction is obtained. A common method of arranging a continued fraction is as follows:

$$\frac{453}{1908} = \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4}}}}}}}$$

A continued fraction is also frequently arranged as follows:

$$\frac{1}{4+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{4}. \qquad (1)$$

The method of determining the different values corresponding to the various parts of a continued fraction will now be explained. All the values for the continued fraction previously given are as follows:

$$\frac{0}{1},\ \frac{1}{4},\ \frac{4}{17},\ \frac{5}{21},\ \frac{14}{59},\ \frac{19}{80},\ \frac{33}{139},\ \frac{151}{636}. \qquad (2)$$

In order to determine these values, which are called "convergents," write the fraction $\frac{0}{1}$ and then the first fraction in the continued fraction; in this case, it is $\frac{1}{4}$. Multiply the numerator of the second fraction, in (2), by the next denominator in the continued fraction, in (1), and add the numerator of the preceding fraction, in (2); thus, $4 \times 1 + 0 = 4$. Then, multiply the denominator of the second fraction, in (2), by the next denominator in the continued fraction, in (1), and add the denominator of the preceding fraction, in (2); thus, $4 \times 4 + 1 = 17$. Write the results as the numerator and denominator of a new fraction, as shown. Multiply the numerator of the fraction last found by the next denominator in the continued fraction and add the preceding numerator to form the numerator of a new fraction; thus $1 \times 4 + 1 = 5$. Do likewise with the denominators; thus, $1 \times 17 + 4 = 21$. Proceed in this manner with the remaining denominators in the continued fraction. The last fraction is equal to the original fraction when reduced to its lowest terms. The convergents following $\frac{0}{1}$ are each nearer in value to the original fraction than any preceding one.

How Continued Fractions are applied to Change-gear Calculations. — Suppose it were desired to calculate the change-gears for a lathe to cut, say, 14.183 threads per inch. It is assumed that the lead-screw has four threads per inch and the lathe screw constant is also 4. What combination of gearing is required for this odd fractional pitch? The true pitch required is $\frac{1}{14.183} = 0.0705+$ inch.

In the calculations, it will be assumed that the pitch should be within the nearest ten-thousandth of an inch; then the gears must be so selected that the pitch of the screw will be 0.0705 ± 0.00005 inch. In other words, it must be greater than 0.07045 and less than 0.07055.

The second step is to convert the decimal into a continued fraction. Thus,

$$0.183 \text{ or } \frac{183}{1000} = \frac{1}{5+}\,\frac{1}{2+}\,\frac{1}{6+}, \text{ etc.}$$

Forming the various convergents, we obtain $\frac{1}{5}$, $\frac{2}{11}$, $\frac{13}{71}$, etc. Now if the second convergent is used, the pitch will be equivalent to 1 $\frac{1}{14\frac{2}{11}}$ and $\frac{1}{14\frac{2}{11}} = \frac{11}{156} = 0.07051+$ inch, which is within the limits. Therefore, since the lathe screw constant is 4 for this particular lathe, the ratio of the change-gears is:

$$\frac{4}{14\frac{2}{11}} = \frac{44}{156} = \frac{22}{78}.$$

Hence, the gear on the lead-screw should have either 156 or 78 teeth, and the gear meshing with it either 44 or 22 teeth.

If the third convergent be used, the pitch will be equivalent to $\frac{1}{14\frac{13}{71}} = 0.07050+$ inch, which, as will be observed, is more accurate. The ratio of the gears in this case is $\frac{4}{14\frac{13}{71}} = \frac{284}{1007}$. Since this fraction cannot be reduced any further, and as it is impracticable to make a gear having 1007 teeth, this ratio is valueless for a simple geared lathe. It can be used, however, with a compound geared lathe, since

$$\frac{284}{1007} = \frac{4}{19} \times \frac{71}{53} = \frac{16}{76} \times \frac{71}{53}.$$

The gears having 16 and 71 teeth are the driving gears and those having 76 and 53 teeth are the driven gears. That this is the correct gear combination may easily be proved by applying the formula previously given for determining the number of threads per inch that may be obtained with any combination of gearing. Thus, in this case,

$$\text{Threads per inch} = \frac{4 \times 76 \times 53}{16 \times 71} = 14.183.$$

Change-gears for Cutting a Worm Thread. — To further illustrate the use of continued fractions, assume that a single-threaded worm is to mesh with a worm-gear of 0.7854 inch circular pitch. What change-gears should be used when cutting the worm thread if the lead-screw has a lathe screw constant of 4?

The linear pitch of the worm is equal to the circular pitch of the worm-gear, or 0.7854 inch. Therefore, the worm has $1 \div 0.7854 = 1.273$ threads per inch. The decimal 0.273 is next converted into a continued fraction. Thus, 0.273 or $\frac{273}{1000} = \frac{1}{3+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1}$, etc. The convergents corresponding to these continued fractions are $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{7}$, $\frac{3}{11}$, etc. If the third convergent is used, the pitch will be equivalent to $\frac{1}{1\frac{2}{7}} = \frac{7}{9}$ $= 0.7777+$, which is within 0.0077 of the required pitch $(0.7854 - 0.7777 = 0.0077)$.

If the fourth convergent is used, $\frac{1}{1\frac{3}{11}} = \frac{11}{14} = 0.7857$, which is within 0.0003 of the required pitch. Since the lathe screw constant is 4, the ratio of the change-gears is represented by $\frac{4}{1\frac{3}{11}} = \frac{44}{14} = \frac{88}{28}$. Therefore, the driving gear or the gear on the stud must have 88 teeth and the driven gear or the one on the lead-screw, 28 teeth. If a compound train of gears were used instead, the driving gears would have 60 and 66 teeth, respectively, and the driven gears, 30 and 42 teeth, respectively. Thus,

$$\frac{44}{14} = \frac{4 \times 11}{2 \times 7} = \frac{60 \times 66}{30 \times 42}.$$

Verifying the accuracy of the foregoing calculations:

$$\text{Threads per inch} = \frac{4 \times 30 \times 42}{60 \times 66} = 1.273, \text{ nearly.}$$

Simple Method of Calculating Change-gears for Worms of Given Diametral Pitch. — The change-gears required for cutting worms that are to mesh with gears of a given diametral pitch may be calculated by the following simple method:

To start with the simplest case, suppose it is desired to cut a worm to mesh with a worm-wheel of one diametral pitch, and assume, for the sake of illustration, that the lathe has a lead-screw with one thread per inch, or a screw of one inch pitch. The circular pitch of a one diametral pitch gear is 3.1416 inches, and a very close approximation to this decimal is the fraction $\frac{22}{7}$. The worm, therefore, must have a pitch of 3.1416 inches, or $\frac{22}{7}$ inches. This fraction, $\frac{22}{7}$, should be memorized, as it is the only number that one need have at hand in making change-gear calculations for worm threads. In the lathe mentioned, the lead-screw would advance the tool one inch for every revolution of the screw and would thus have to turn 3.1416 times to advance the tool 3.1416 inches. In other words, the lead-screw should turn 3.1416 times as fast as the worm which is being cut. As the fraction $\frac{22}{7}$ is nearly equivalent to 3.1416, this result would be accomplished with a 22-tooth gear on the stud and a 7-tooth pinion on the lead-screw, or gears in this ratio, say, 44 on the stud and 14 on the screw.

Now suppose that instead of a lead-screw of one thread per inch, there was one with three threads per inch. The screw would then have to turn three times as fast as before, to accomplish which the gear on the stud should be three times as large, or else the gear on the screw three times as small as before. The gearing should be in the ratio, therefore, of $3 \times \frac{22}{7}$, or $\frac{66}{7}$. If the lead-screw had four threads per inch, the gearing should be in the ratio of $\frac{88}{7}$, and if six threads, in the ratio of $\frac{132}{7}$. Assume, finally, that it is desired to cut a worm of some pitch other than one, say, to mesh with a worm-wheel of four diametral pitch. The threads of the worm would then be four times as near together as before and the lead-screw should turn only one-fourth as fast. The gear on the lead-screw could thus be four times as large as in the first example, or the gear on the stud four times as small. With a three-pitch lead-screw, the gearing should be in the ratio of $\frac{66}{28}$; with the four-pitch screw, $\frac{88}{28}$; and with the six-pitch screw, $\frac{132}{28}$. If a worm for a six-pitch worm-wheel were desired, then for the three cases the gearing would be in the ratio of $\frac{66}{42}$, $\frac{88}{42}$, and $\frac{132}{42}$, respectively. Of course, it is understood that these fractions may be reduced to lower terms, in order to reduce the size of the gears. Thus, for a three-pitch lead-screw and four-pitch worm, the ratio $\frac{66}{28}$ was found, but gears having 33 and 14 teeth, respectively, could be used instead.

The foregoing calculations apply only to lathes in which the spindle and lead-screw make the same number of turns with equal gears. Where the spindle turns faster or more slowly than the lead-screw, with equal change-gears, allowance must be made for this. With compound-geared lathes, the simplest plan is to use equal intermediate gears and to put gears on the stud and screw, as calculated above. The following general rule may be used for finding the change-gears to use:

Rule: Multiply the lathe screw constant by 22 to obtain the number of teeth in the stud gear and multiply the diametral pitch of the worm-wheel (for which the worm is intended) by 7, to obtain the number of teeth in the gear on the lead-screw.

Example. — Determine what change-gears should be used for cutting a thread on a worm to mesh with a worm-wheel of 6 diametral pitch, assuming that the lead-screw of the lathe has 4 threads per inch, and the lathe spindle and stud are geared in the ratio of 1 to 1, so that the lathe screw constant is 4.

Applying the foregoing rule,

$$\frac{4 \times 22}{6 \times 7} = \frac{88}{42}.$$

The gear on the stud should have 88 teeth and the gear on the lead-screw, 42 teeth. If these gears are used, the error in the lead of the worm will be very slight. This error is due to the fact that $\frac{22}{7}$ is not quite equal to 3.1416, as previously explained. The number of threads per inch cut with the gears referred to equals $\frac{4 \times 42}{88} = 1.909$. Hence, the lead of the worm thread equals $\frac{1}{1.909} = 0.5238$ inch. The correct lead of a worm to mesh with a worm-wheel of 6 diametral pitch is 0.5236 inch; therefore, the error in lead equals $0.5238 - 0.5236 = 0.0002$ inch.

Change-gears for Milling Spirals. — The method for the figuring of change-gears for cutting spirals on the milling machine is, in principle, exactly the same as that used for figuring change-gears for the lathe.

In Fig. 2 is shown an end view of an index-head for a milling machine, placed on the top of the milling machine table. At *A* is shown the end of the table feed-screw, and *B* is a gear placed on this feed-screw. This gear is commonly called the feed-screw gear, and it imparts motion, through an intermediate gear *H*, to gear *C* which is placed on the stud *D*; from this stud, in turn, motion is imparted by gearing to the worm of the index-head and from the worm to the worm-wheel mounted on the index-head spindle. Thus, when connected by gearing in this manner, the index-head spindle may be rotated from the feed-screw. The gear *C* on the stud *D* is called the "worm-gear"; this worm-gear should not be confused with the worm-wheel which is permanently attached to the index-head spindle.

Simple gearing is shown in Fig. 2, while in Fig. 3 the gears are compounded. In this case, *B* still represents the feed-screw gear, while *E* is the gear on the intermediate stud which meshes with *B*, and *F* is the second gear on the same intermediate stud, meshing with gear *C*. The object of the calculation is to find the numbers of teeth in gears *B* and *C* which are used in a simple train, as in Fig. 2; or in the gears *B*, *E*, *F*, and *C* as used in a compound train of gears, as shown in Fig. 3.

The Lead of a Milling Machine. — If gears with an equal number of teeth are placed on the feed-screw *A* and the stud *D* in Fig. 2, then the lead of the milling machine is the distance the table will travel while the index-spindle makes one complete revolution. This distance is a constant used in figuring the change-gears, and may vary for different milling machines.

The lead of a helix or spiral is the distance, measured along the axis of the work, in which the spiral makes one full turn

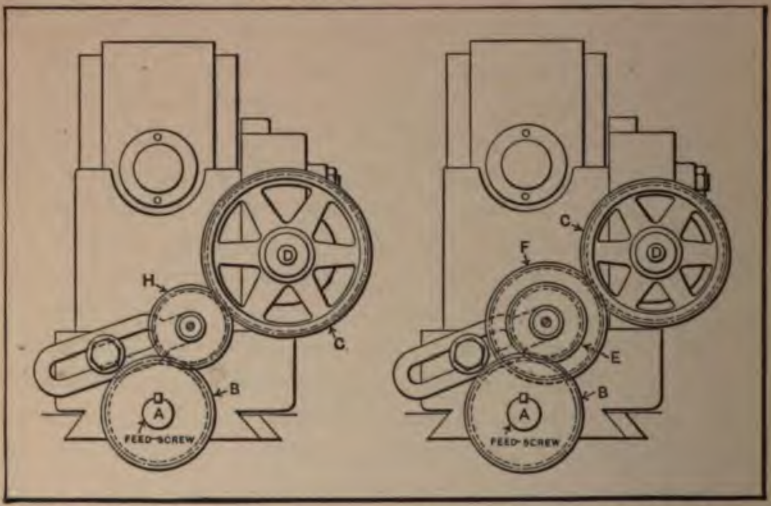


Fig. 2. Simple Gearing for Dividing-head

Fig. 3. Compound Gearing for Dividing-head

around the work. The lead of the milling machine may, therefore, also be expressed as the lead of the spiral that will be cut when gears with an equal number of teeth are placed on studs *A* and *D*, and an idler of suitable size interposed between the gears.

Rule: To find the lead of a milling machine, place equal gears on stud *D*, and on feed-screw *A*, Fig. 2, and multiply the number of revolutions made by the feed-screw to produce one revolution of the index-head spindle, by the lead of the thread on the feed-screw.

This rule may be expressed as a formula:

$$\text{Lead of milling machine} = \begin{matrix}\text{rev. of feed-screw for one} \\ \text{revolution of index-spindle} \\ \text{with equal gears}\end{matrix} \times \begin{matrix}\text{lead of} \\ \text{feed-screw}\end{matrix}$$

Assume that it is necessary to make 40 revolutions of the feed-screw to turn the index-head spindle one complete revolution, when the gears *B* and *C*, Fig. 2, are equal, and that the lead of the thread on the feed-screw of the milling machine is $\frac{1}{4}$ inch; then the lead of the machine equals:

$$40 \times \tfrac{1}{4} \text{ inch} = 10 \text{ inches.}$$

Change-gears for Spiral of Given Lead. — As has already been stated, the lead of the machine means the distance which the table of the milling machine moves forward in order to turn the work placed on the index-head spindle one complete revolution when change-gears with an equal number of teeth are used. If then, for instance, a spiral is to be cut, the lead of which is twice as long as the lead of the machine, change-gears of such a ratio must be used that the index-head spindle will turn only one-half a revolution while the table moves forward a distance equal to the lead of the machine.

Assume that it is desired to cut a spiral having a lead of 20 inches, that is, making one complete turn in a length of 20 inches, and that the lead of the milling machine is 10 inches. Then the ratio between the speeds of the feed-screw and of stud *D* must be 2 to 1, which means that the feed-screw, which is required to turn twice while stud *D* turns once, must have a gear that has only one-half the number of teeth of the gear placed on stud *D*. If the lead of the machine is 10 inches and the lead of the spiral required to be cut on a piece of work is 30 inches, then the ratio between the speed of the gears would be 3 to 1, which is the same as the ratio between the lead of the spiral to be cut to the lead of the machine. (30 to 10 = 3 to 1, or, as it is commonly written, 30 : 10 = 3 : 1.)

The rule for finding the change-gears can be expressed by a simple formula:

$$\frac{\text{Lead of spiral to be cut}}{\text{Lead of milling machine}} = \frac{\text{number of teeth in gear on worm stud } (D, \text{ Fig. } 2)}{\text{number of teeth in gear on feed-screw}}.$$

Rule: To find the change-gears to be used in a simple train of gearing when cutting spirals in the milling machine, place the lead of the spiral as the numerator and the lead of the milling machine as the denominator of a fraction, and multiply the numerator and the denominator by the same number until a new fraction is obtained in which the numerator and denominator give suitable numbers of teeth for the gears.

Example.— A milling machine has 4 threads per inch on the feed-screw and 40 revolutions of the feed-screw are necessary to make the index spindle turn one complete revolution when gears *B* and *C*, Fig. 2, are equal. What change-gears are required on this machine to cut a spiral, the lead of which is 12 inches?

First find the lead of the machine. As the feed-screw is single-threaded and has 4 threads per inch, the lead of the screw thread is $\frac{1}{4}$ inch and the lead equals:

$$40 \times \tfrac{1}{4} \text{ inch} = 10 \text{ inches} = \text{lead of machine.}$$

To find the change-gears, place the lead of the required spiral as the numerator of a fraction and the lead of the machine as the denominator, and multiply both the numerator and the denominator by the same number until a new fraction is obtained in which the numerator and denominator express suitable numbers of teeth. Following this rule, then,

$$\frac{12}{10} = \frac{12 \times 4}{10 \times 4} = \frac{48}{40}.$$

The gear with 48 teeth is placed on stud *D* which is required to revolve more slowly than the lead-screw, in order to cut a spiral which is 12 inches, when the spiral cut with equal gears is only 10 inches. The gear having 40 teeth is placed on the feed-screw. An intermediate gear is put between the gear on the feed-screw and the gear on stud *D*; the number of teeth in this intermediate gear has no influence on the speed ratio of feed-screw *A* and stud *D*, but simply serves to transmit motion from one gear to the other.

If it is not possible to find a set of two gears that will transmit the required motion, it is necessary to use compound gearing. In this case, the manner in which the gears are found is exactly the same as the method used for compound gearing in the lathe.

As an example, assume that the lead of a machine is 10 inches, and that a spiral having a 48-inch lead is to be cut. Following the method previously explained, then,

$$\frac{48}{10} = \frac{6 \times 8}{2 \times 5} = \frac{(6 \times 12) \times (8 \times 8)}{(2 \times 12) \times (5 \times 8)} = \frac{72 \times 64}{24 \times 40}.$$

The gear having 72 teeth is placed on the stud *D* and meshes with the 24-tooth gear *F* (see Fig. 3), on the intermediate stud. On the same intermediate stud is then placed the gear having 64 teeth, which is driven by the gear having 40 teeth placed on the feed-screw. This makes the gears having 72 and 64 teeth the driven gears, and the gears having 24 and 40 teeth the driving gears, the whole train of gears being driven from the feed-screw of the table.

In general, for compound gearing, the following formula may be used:

$$\frac{\text{Lead of spiral to be cut}}{\text{Lead of machine}} = \frac{\text{product of driven gears}}{\text{product of driving gears}}.$$

Figuring Change-gears when Lead of Spiral is Fractional. — When the lead of a spiral is not an even or whole number of inches, but fractional or decimal, change-gears may be calculated conveniently by the method to be illustrated, by an example.

Example. — Assume that a spiral (or, more properly, helical) groove having a lead of 1.25 inch is to be milled and that the lead of the machine is 10. What combination of change-gears should be used?

The ratio of the gearing is equivalent to $\frac{1.25}{10}$. If both terms are multiplied by 100, the expression is changed to $\frac{125}{1000}$. Resolving this expression into factors,

$$\frac{125}{1000} = \frac{5 \times 25}{10 \times 100} = \frac{30 \times 25}{60 \times 100} = \frac{24 \times 25}{48 \times 100} = \frac{36 \times 24}{72 \times 96}.$$

Therefore, gears having 30 and 25 teeth can be used as the two driven gears when gears of 60 and 100 teeth are used as drivers, or any other combinations may be used.

Example. — Assume that a spiral groove is to be milled having a lead of 2.22 inches, and that the lead of the machine is 10. Find the gears that may be used.

By applying the method previously referred to, we have

$$\frac{222}{1000} = \frac{6 \times 37}{10 \times 100} = \frac{36 \times 37}{60 \times 100} = \frac{24 \times 37}{40 \times 100} = \frac{24 \times 74}{80 \times 100}.$$

There are also several other combinations which might be used to obtain a lead of 2.22 inches, the numbers above the line representing the driven gears and those below the line, the driving gears. If the lead should be given in thousandths, both terms may be multiplied by 1000, or the required lead may be written down as a whole number, as many ciphers being annexed to the 10 in the denominator as there are decimal places in the required lead. For instance, if the lead is 2.176 inches, the ratio of the gears is $\frac{2176}{10,000}$.

CHAPTER X

ANGLES AND THE USE OF TABLES WHEN FIGURING ANGLES

THERE is no branch of mathematics which is of greater importance and practical value to machinists and toolmakers than that which deals with angles, and especially with the solution of triangles. A general knowledge of this subject makes it possible to perform operations readily which, without

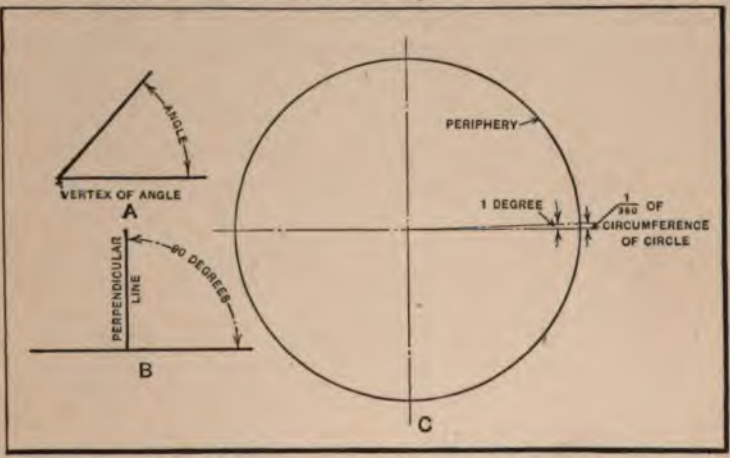


Fig. 1. Diagrams illustrating Angular Measurement

calculation, would either be impossible or require a long and tedious cut-and-try method. Problems requiring the solution of triangles are common in connection with many operations on the milling machine, and especially in toolmaking practice. This chapter explains the meanings of the various functions of angles such as the sine, cosine, tangent, etc., and then deals with the tables which are used to obtain the numerical values for different angles and functions. The practical application of these tables will be illustrated in Chapters XI and XII.

Angular Measurement. — When two lines meet as shown at *A* in Fig. 1, they form an angle with each other. The point where the two lines meet or intersect is called the *vertex* of the angle. The two lines forming the angle are called the *sides* of the angle. Angles are measured in degrees and subdivisions of a degree. If the circumference (periphery) of a circle is divided into 360 parts, each part is called *one degree*, and the angle between two lines from the center to the ends of the small part of the circle is a one-degree angle, as shown at *C*. As the whole circle contains 360 degrees, one-half of a circle contains 180 degrees, and one-quarter of a circle, 90 degrees.

In order to obtain finer subdivisions than the degree for the measurement of angles, one degree is divided into 60

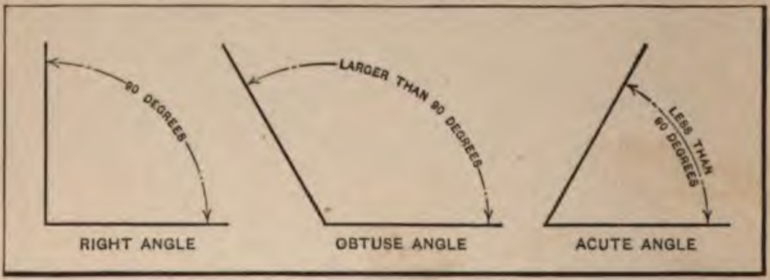


Fig. 2. Names given to Angles

minutes, and one minute into 60 seconds. Any part of a degree can be expressed in minutes and seconds, for instance, $\frac{1}{2}$ degree = 30 minutes; $\frac{1}{3}$ degree = 20 minutes; and since $\frac{1}{4}$ degree = 15 minutes, $\frac{3}{4}$ degree = 45 minutes. In the same way, $\frac{1}{2}$ minute = 30 seconds; $\frac{1}{3}$ minute = 20 seconds; $\frac{1}{4}$ minute = 15 seconds; and $\frac{3}{4}$ minute = 45 seconds.

The word degree is often abbreviated "deg.," or the sign (°) is used to indicate degrees; thus, $60° = 60$ degrees. In the same way, $60' = 60$ minutes (min.), and $60'' = 60$ seconds (sec.).

A 90-degree angle is called a *right* angle. An angle larger than 90 degrees is called an *obtuse* angle, and an angle less than 90 degrees is called an *acute* angle. (See Fig. 2.) Any angle which is not a right angle is called an *oblique* angle.

When two lines form a right or 90-degree angle with each other, as shown at *B* in Fig. 1, one line is said to be *perpendicular* to the other.

Angles are said to be equal when they contain the same number of degrees. The angle at *A*, Fig. 3, and the angle at *B* are equal, because they are both 60 degrees; that the sides of the angle at *B* are longer than the sides of the angle at *A*

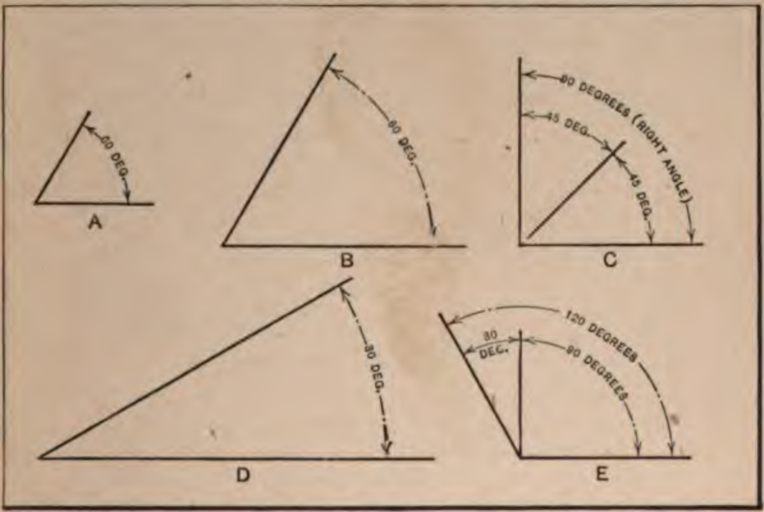


Fig. 3. Diagrams illustrating Angles of 30, 45, 60, 90, and 120 Degrees

has no influence on the angle, because of the fact that an angle is only the *difference in direction* of two lines. The angle at *D*, which contains only 30 degrees, is only one-half of the angle at *A*. One-half of a right angle is 45 degrees, as shown at *C*. At *E* is shown an angle which is 120 degrees, and which can be divided into a right or 90-degree angle, and a 30-degree angle.

Functions of Angles. — The object of that part of mathematics called "trigonometry" is to furnish the methods by which the unknown sides and angles in a triangle may be determined when certain of the sides and angles are given.

The sides and angles of any triangle, which are not known, can be found when (1) all three sides, (2) two sides and one angle, or (3) one side and two angles are given. In other words, if the triangle is considered as consisting of six parts, three angles, and three sides, the unknown parts can be determined when any three of the parts are given, provided at least one of the given parts is a side.

In order to introduce the values of the angles in calculations of triangles, use is made of certain expressions called *trigonometrical functions* or *functions of angles.* The names of

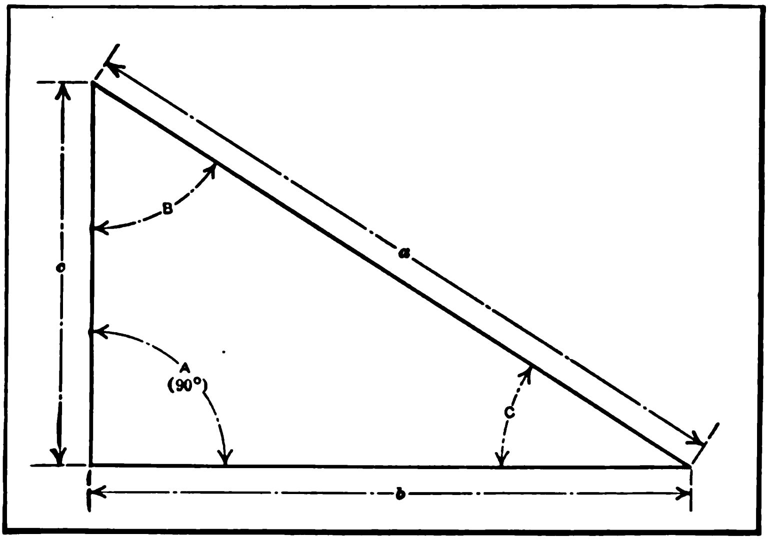


Fig. 4. Right-angled Triangle

these expressions are: *sine, cosine, tangent, cotangent, secant, and cosecant,* and they are usually abbreviated as follows:

sin = sine,	cot = cotangent,
cos = cosine,	sec = secant,
tan = tangent,	cosec = cosecant.

In Fig. 4 is shown a right-angled triangle. The lengths of the three sides are represented by a, b, and c, respectively, and the angles opposite each of these sides are marked A,

B, and C, respectively. Angle A is the right angle in the triangle. The side a opposite the right angle is the *hypotenuse*. The side b is called the side *adjacent* to the angle C, but is, of course, also the side *opposite* to angle B. In the same way, the side c is called the side adjacent to angle B, and the side opposite to angle C. The reason for these names is made clear by studying the figure.

The meanings of the various functions of angles previously named can be explained by the aid of a right-angled triangle.

The sine of an angle equals the opposite side divided by the hypotenuse. The sine of angle B thus equals the side b, which is opposite to the angle, divided by the hypotenuse a. Expressed as a formula:

$$\sin B = \frac{b}{a}.$$

If $a = 16$, and $b = 9$, then $\sin B = \frac{9}{16} = 0.5625$.

The cosine of an angle equals the adjacent side divided by the hypotenuse. The cosine of angle B thus equals the side c, which is adjacent to this angle, divided by the hypotenuse a, or, expressed as a formula:

$$\cos B = \frac{c}{a}.$$

If $a = 24$, and $c = 15$, then $\cos B = \frac{15}{24} = 0.625$.

The tangent of an angle equals the opposite side divided by the adjacent side. The tangent of angle B thus equals the side b divided by side c, or:

$$\tan B = \frac{b}{c}.$$

If $b = 28$, and $c = 25$, then $\tan B = \frac{28}{25} = 1.12$.

The cotangent of an angle equals the adjacent side divided by the opposite side. The cotangent of angle B thus equals the side c divided by the side b, or:

$$\cot B = \frac{c}{b}.$$

If $b = 28$, and $c = 25$, then $\cot B = \frac{25}{28} = 0.89286.$

The secant of an angle equals the hypotenuse divided by the adjacent side. The secant of angle B thus equals the hypotenuse a divided by the side c adjacent to the angle, or:

$$\sec B = \frac{a}{c}.$$

If $a = 24$, and $c = 15$, then $\sec B = \frac{24}{15} = 1.6.$

The cosecant of an angle equals the hypotenuse divided by the opposite side. The cosecant of angle B thus equals the hypotenuse a divided by the side b opposite the angle, or:

$$\operatorname{cosec} B = \frac{a}{b}.$$

If $a = 16$, and $b = 9$, then $\operatorname{cosec} B = \frac{16}{9} = 1.77778.$

The rules given above are easily memorized, and the student should go no further before he can see at a glance the various functions in a given right-angled triangle.

If the functions of the angle C were to be found instead of the functions of angle B, as given above, they would be as follows:

$$\sin C = \frac{c}{a} \qquad \cos C = \frac{b}{a} \qquad \tan C = \frac{c}{b}$$

$$\cot C = \frac{b}{c} \qquad \sec C = \frac{a}{b} \qquad \operatorname{cosec} C = \frac{a}{c}$$

It must be remembered that the functions of the angles can be found in this manner only when the triangle is right-angled. If the triangle has the shape shown by the full lines in Fig. 5, the sine of angle D, for instance, cannot be expressed by any relation between two sides of this triangle. The sine of angle D, however, can be found by constructing a right-angled triangle by extending the side e to the point P, from where a line can be drawn at right angles to the vertex or point of angle *E*, as shown by the dotted line. The sine of angle *D* would then be the length of the dotted line *g* divided by the length of the line *h*, these two lines being, respectively, the side opposite angle *D*, and the hypotenuse, in a right-angled triangle. In the same way, the tangent of angle *D* would be the side *g* divided by the side *f*.

Finding the Values of the Functions of Angles. — In Fig. 6 is shown a right-angled triangle where the side opposite angle

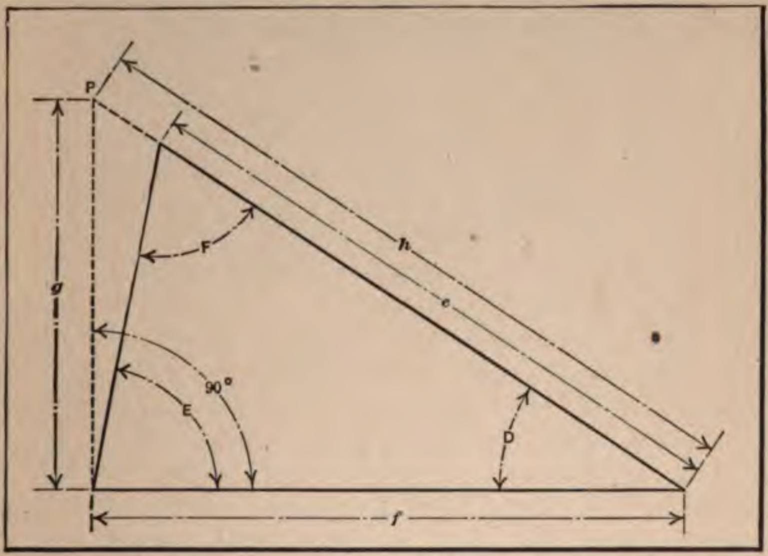


Fig. 5. Acute-angled Triangle (shown by full lines)

B is four inches, the side opposite angle *C* is 3 inches, and the hypotenuse is 5 inches. Find the values of the functions of the angles *B* and *C*.

Following the rules previously given for finding the sine, cosine, tangent, etc.:

$\sin B = \frac{4}{5} = 0.8$ $\cos B = \frac{3}{5} = 0.6$

$\tan B = \frac{4}{3} = 1.333$ $\cot B = \frac{3}{4} = 0.75$

$$\sec B = \frac{5}{3} = 1.667 \qquad \text{cosec } B = \frac{5}{4} = 1.25$$

The functions for angle C are as follows:

$$\sin C = \frac{3}{5} = 0.6 \qquad \cos C = \frac{4}{5} = 0.8$$

$$\tan C = \frac{3}{4} = 0.75 \qquad \cot C = \frac{4}{3} = 1.333$$

$$\sec C = \frac{5}{4} = 1.25 \qquad \text{cosec } C = \frac{5}{3} = 1.667$$

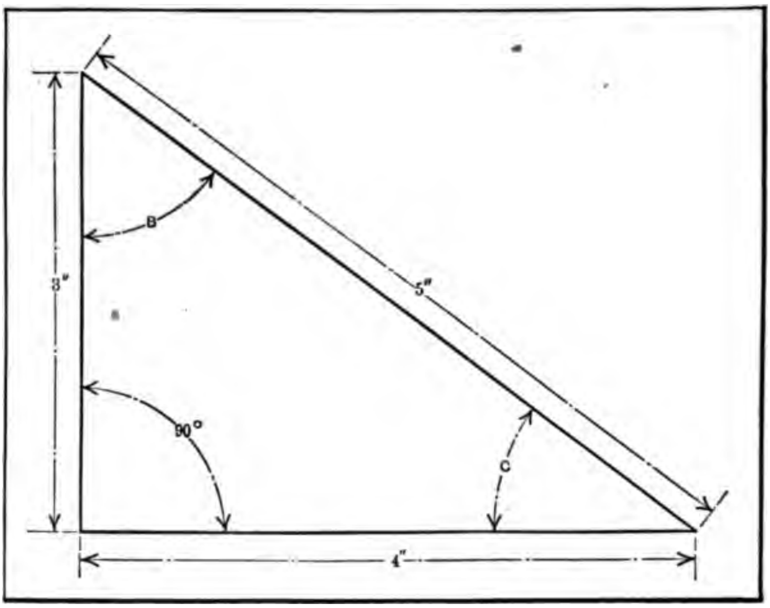


Fig. 6. Right-angled Triangle having a 4-inch Base, 3-inch Altitude, and 5-inch Hypotenuse

The secant and cosecant, being merely the values of 1 divided by the cosine and sine, are not often used in calculations, or included in tables of angular functions.

By studying the results obtained in the calculations above it will be noted that in a right-angled triangle there is a definite relation between the functions of the two acute angles. The *sine of angle B* equals the cosine of angle C; the tangent of angle B equals the cotangent of angle C, etc. This is true of all right-angled triangles.

As the sum of the three angles in a triangle always equals 180 degrees, and as a right angle equals 90 degrees, it follows that the sum of the two acute angles in a right-angled triangle equals $180 - 90 = 90$ degrees. The angle B (Fig. 6) which together with angle C forms a 90-degree angle, is called the *complement* of angle C. In the same way, angle C is the complement of angle B. When any two angles together make 90 degrees, the one is the complement of the other, and in all such cases, the sine of the one equals the cosine of the other, and vice versa, the tangent of the one equals the cotangent of the other, etc.

Tables of the Functions of Angles. — In solving problems which pertain to angles, tables of trigonometric functions which give the numerical values for different angles and functions are used. The sine of angle C, Fig. 6, equals $3 \div 5 = 0.6$. Now, in order to determine the value of angle C in degrees, a table of sines is referred to. Such a table shows what angle corresponds to sine 0.6. If, on the other hand, angle C and the length of the hypotenuse were known and the problem were to determine the length of the side opposite, the sine of angle C would be found first by referring to a table, and then this sine would be multiplied by the hypotenuse, which is 5 in this case, to obtain the length of the side opposite.

When using formulas of the type

$$A = \frac{16 \times \sin 36 \text{ degrees}}{2},$$

it is not possible, of course, to find the value of A unless there is some means of transforming the expression "sin 36 degrees" (read sine of 36 degrees) into plain figures. In other words, the numerical value of "sine 36 degrees" must be known before the value of A can be determined. If sine 36 degrees equals 0.58779, then, by inserting this value in the formula, it reads:

$$A = \frac{16 \times 0.58779}{2} = 4.70232.$$

The numerical values for the natural or trigonometric functions, which must be known before a formula containing an expression with a trigonometric function can be calculated, are given in tables which are found in engineering handbooks. These tables are not all arranged in exactly the same way and some are more complete than others. The accompanying tables give the values of the sines, cosines, tangents, and cotangents for all degrees and for every 10 minutes or one-sixth of a degree. The tables in MACHINERY'S HANDBOOK give the values for all degrees and minutes and they include values for all of the functions. The accompanying tables, however, will serve to illustrate how tables of functions are used and they are complete enough for many practical problems. By means of such tables, when the angle is given, the angular function can be found, and when the function is given, the angle can be determined.

At the top of Tables 1 and 2, the heading reads "Table of Sines," and at the bottom is the legend "Table of Cosines." At the top of Tables 3 and 4, the heading reads "Table of Tangents," and at the bottom is the legend "Table of Cotangents." At the top of all the tables, the heading of the extreme left-hand column reads "Deg.," and the following columns are headed 0', 10', 20', etc. At the bottom of the tables the same legends are placed under the columns, but reading from right to left.

When the sine or tangent of a given angle is to be found, first find the number of degrees in the extreme left-hand column in the respective tables, and then locate the number of minutes at the top of the table. Then follow the column, over which the number of minutes is given, downward until arriving at the figure in line with the given number of degrees. This figure is the numerical value of the sine or tangent for the given angle. If the angle is given in even degrees with no minutes, the corresponding function will be found opposite the number of degrees in the column marked 0' at the top.

The cosines and cotangents of angles are found in the same *tables as the* sines and tangents, but the tables in this case are read from the bottom up. The number of degrees is found in the extreme right-hand column and the number of minutes at the bottom of the columns. If the number of minutes given is not an even multiple of 10, as 10′, 20′, 30′, etc., but, say 27′, it is generally near enough to take the figures for the nearest number of minutes given, being in this case for 30′.

Examples illustrating the Use of the Tables. — *Example* 1. — Find from the tables given the sine of 56 degrees, or, as it is written in formulas, sin 56°. — The sines are found by reading Tables 1 and 2 from the top; the number of degrees, 56, is found in Table 2 in the left-hand column, and opposite 56 in the column 0′, read off 0.82903.

Example 2. — Find sin 56° 20′. — In the column marked 20′ at the top, follow downward until opposite 56 in the left-hand column. The value 0.83227 is the sine of 56° 20′.

Example 3. — Find cos 36° 20′. — To find the cosines, read the tables from the bottom, and locate 36 in the right-hand column in Table 2. Then follow the column marked 20′ at the bottom upward until opposite 36, and read off 0.80558.

Example 4. — Find tan 56° 40′. — The tangents are found in Tables 3 and 4 by locating the number of degrees in the left-hand column and reading the value in the column under the specified number of minutes. In Table 4, then, tan 56° 40′ is found to be 1.5204.

Example 5. — Find the cotangent of 56° 40′. — Read the tables from the bottom, locating 56 in the right-hand column, and find the required value in line with this figure in the column marked 40′ at the bottom. Thus, cot 56° 40′ = 0.65771.

Example 6. — Find sin 20° 48′. — For shop calculations, it is almost always near enough to find the value of the angular functions for the nearest 10 minutes. Therefore, in this case find sin 20° 50′, which is 0.35565.

Example 7. — The sine for a certain angle, which may be called *a*, equals 0.53238. Find the angle. — In the body of the tables of sines find the number 0.53238. It will be seen that this number is opposite 32 degrees and in the column headed 10′ at the top. The angle *a*, therefore, equals 32° 10′.

1. TABLE OF SINES

Read degrees in left-hand column and minutes at top
Example: sin 7° 10′ = .12475

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35021	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51504	59
31	.51504	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57358	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64279	50
40	.64279	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom
Example: cos 56° 20′ = .55436

2. TABLE OF SINES

Read degrees in left-hand column and minutes at top
Example: sin 56° 20′ = .83227

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85717	.85866	.86014	.86162	.86310	.86457	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88295	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97815	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98481	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	.99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99996	.99998	.99999	1.00000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES

Read degrees in right-hand column and minutes at bottom
Example: cos 7° 10′ = .99218

3. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top
Example: tan 7° 10′ = .12573

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21550	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64528	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	.80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83910	50
40	.83910	.84406	.84906	.85408	.85912	.86419	.86928	49
41	.86928	.87440	.87955	.88472	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92709	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97132	.97699	.98269	.98843	.99419	1.00000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom
Example: cot 56° 20′ = .66607

4. TABLE OF TANGENTS

Read degrees in left-hand column and minutes at top
Example: tan 56° 20′ = 1.5013

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1.2349	39
51	1.2349	1.2422	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS

Read degrees in right-hand column and minutes at bottom
Example: cot 7° 10′ = 7.9530

Example 8. — Cot b equals 0.77195. Find b. — The cotangents are read from the bottom in Tables 3 and 4. The value 0.77195 is located opposite 52 in the right-hand column and in the column marked 20′ at the bottom. Angle b, then, is 52° 20′.

Example 9. — Sin b equals 0.31190. Find b. — It will be found that the value 0.31190 is not given in the table of sines; the nearest value in the table is 0.31178. For shop calculations, it is near enough to consider the angle b equal to the angle corresponding to this latter value; the angle, then, is 18° 10′.

Functions of Angles Greater than 90 Degrees. — The accompanying tables give the angular functions only for angles up to 90 degrees (or 89 degrees 60 minutes, which, of course, equals 90 degrees). In obtuse triangles, one angle, however, is greater than 90 degrees, and the tables can be used for finding the functions for angles larger than 90 degrees also.

The sine of an angle greater than 90 degrees but less than 180 degrees equals the sine of an angle which is the difference between 180 degrees and the given angle.

Example: Sin 118° = sin (180° − 118°) = sin 62°. In the same way, sin 150° 40′ = sin (180° − 150° 40′) = sin 29° 20′.

The cosine, tangent, and cotangent for an angle greater than 90 but less than 180 degrees equals, respectively, the cosine, tangent, and cotangent of the difference between 180 degrees and the given angle, but, in this case, the angular function found has a *negative* value, preceded by a minus sign. (See "Positive and Negative Quantities," Chapter II.)

Example 1. — Find tan 150°.

Tan 150° = −tan (180° − 150°) = −tan 30°. From the tables we have tan 30° = 0.57735; thus tan 150° = −0.57735.

Example 2. — Find sin 155° 50′.

As previously explained, sin 155° 50′ = sin (180° − 155° 50′) = sin 24° 10′ = 0.40939.

Example 3. — Find tan 123° 20′.

Tan 123° 20′ = −tan (180° − 123° 20′) = −tan 56° 40′ = −1.5204.

[In calculations of triangles it is very important that the minus sign should not be omitted in the cosines, tangents, and cotangents of angles between 90 and 180 degrees.]

Finding the Angle when the Function is given. — When the value of the function of an angle is given, and the angle is required in degrees and minutes, the function is located in the tables and the corresponding angle found by a process the reverse of that employed for finding the functions when the angle is given. If the value of the function cannot be found exactly in the tables, use the nearest value found.

Example 1. — The sine of a certain angle, which may be represented by the letter a, equals 0.53238. Find the angle.

The function 0.53238 is located in the first table of sines. When located, the degrees and minutes of the angle are read off directly. If, as in this case, the number (0.53238) represents the sine, then the number of degrees is read off at the left, and the number of minutes at the top, of the column.

Example 2. — The cotangent of an angle is 0.77195. Find the angle.

This value will be found in Table 3, and, as the cotangent is required, the angle is found in the column to the right, and the number of minutes at the bottom, of the column. The required angle is 52 degrees 20 minutes.

Example 3. — The tangent of angle a equals -3.3402. Find a.

The positive value 3.3402 is first located and the corresponding angle found. This angle is 73 degrees 20 minutes. As the tangent is negative (preceded by a minus sign), the angle a is not 73 degrees 20 minutes, but $(180^\circ - 73^\circ\ 20') = 106^\circ\ 40'$

Example 4. — If sine a equals 0.29487, what is the value of angle a?

It will be seen that the function 0.29487 cannot be found exactly in these particular tables. The nearest value to be found in the sine columns is 0.29515, which shows that the angle is nearly 17 degrees 10 minutes.

CHAPTER XI

SOLUTION OF RIGHT-ANGLED TRIANGLES

If the lengths of two sides of a right-angled triangle are known, the third side can be found by a simple calculation. In every right-angled triangle the hypotenuse equals the square root of the sum of the squares of the two sides forming the right angle. If the hypotenuse equals a, and the sides forming the right angle, b and c, respectively, as shown in Fig. 1, then:

$$a = \sqrt{b^2 + c^2} \quad (1)$$

Each of the sides b and c can also be found if the hypotenuse and one of the sides are known. The following formulas would then be used:

$$b = \sqrt{a^2 - c^2} \quad (2)$$

$$c = \sqrt{a^2 - b^2} \quad (3)$$

Example. — Assume that side b is 18 inches, and side c, 7.5 inches. What is the length of the hypotenuse a?

If the values of b and c are inserted in the formula given above for a, then:

$$a = \sqrt{18^2 + 7.5^2} = \sqrt{18 \times 18 + 7.5 \times 7.5} = \sqrt{324 + 56.25}$$
$$= \sqrt{380.25} = 19.5.$$

Assume that the length of the hypotenuse is 10 inches and that the side c is 6 inches. What is the length of the side b?

Using the formula given above for b, and inserting the values of a and c, then:

$$b = \sqrt{10^2 - 6^2} = \sqrt{10 \times 10 - 6 \times 6} = \sqrt{100 - 36} = \sqrt{64} = 8.$$

Thus, whenever two sides of a right-angled triangle are given, the third side can always be found by a simple arithmetical calculation. To find the angles, however, it is necessary to use the tables of sines, cosines, tangents, and cotangents; and if only one side and one of the acute angles are given, the tables of functions must be used for finding the lengths of the other sides, as explained in the following.

Use of Functions of Angles for Solving Right-angled Triangles. — In every right-angled triangle, one angle, the right or 90-degree angle, is, of course, always known. In a right triangle, therefore, the unknown sides and angles can be found when either two sides, or one side and one of the acute angles, are known. The methods of solution of right-angled

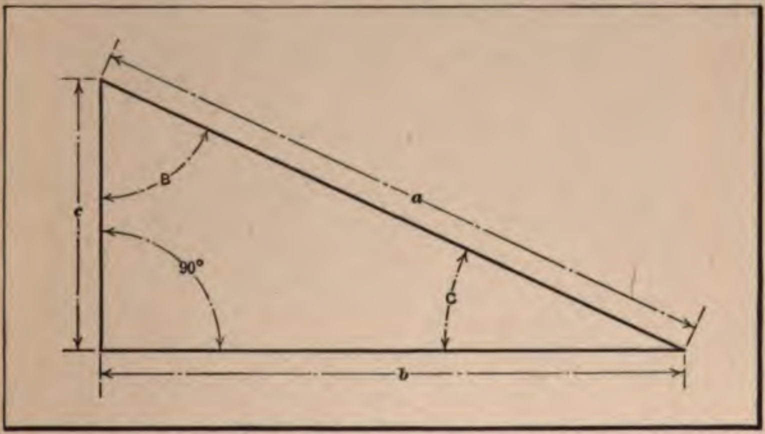


Fig. 1. Right-angled Triangle with Letters Indicating Values used in Calculations

triangles may be divided into four classes, according to which sides and angles are given or known:

1. Two sides known.
2. The hypotenuse and one acute angle known.
3. One acute angle and its adjacent side known.
4. One acute angle and its opposite side known.

Case 1. — When two sides are known, the third side is found by one of the preceding formulas (1), (2), or (3).

In these formulas, a is the hypotenuse, and b and c the sides forming the right angle.

The acute angles B and C, Fig. 1, are found by determining either the sine, cosine, tangent, or cotangent for the angles, and obtaining the angles, expressed in degrees and minutes, from the trigonometric tables. When one angle has been found, the other can be found directly without reference to the tables, because the sum of the acute angles in a right-angled triangle equals 90 degrees, and if one of them is known, the other must equal 90 degrees minus the known angle. Expressed as formulas these would be:

$$B = 90° - C; \qquad C = 90° - B.$$

Example 1. — Assume that the hypotenuse a (Fig. 1) of a right-angled triangle is 5 inches and side b is 4 inches. Find angles B and C and the length of side c.

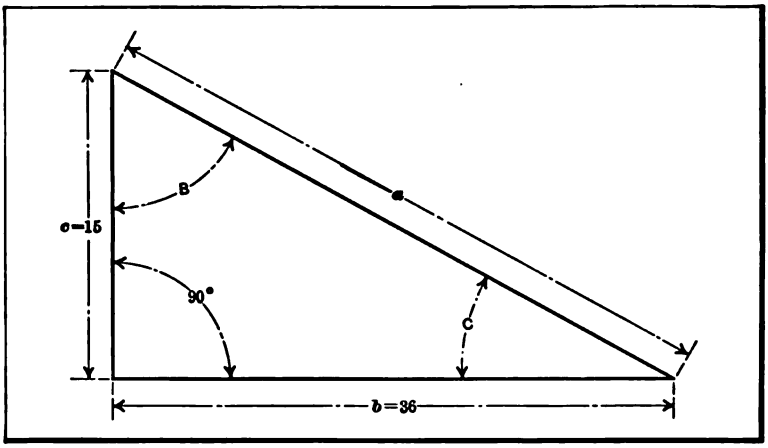


Fig. 2. When the Base *b* and the Altitude *c* are given, to Find the Hypotenuse of *a* and Angles *B* and *C*

The side c is first found by Formula (3), a and b being inserted in this formula as below:

$$c = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3.$$

The side opposite an angle divided by the hypotenuse gives the sine of the angle.

$$\sin C = \frac{3}{5} = 0.6.$$

By referring to the trigonometric tables, it will be found that the nearest value to 0.6 in the columns of sines is 0.59948, and the angle corresponding to this value is 36° 50′. Angle C, then, equals 36° 50′, nearly.

In the same way,

$$\sin B = \frac{4}{5} = 0.8.$$

From the tables it is found that the nearest value in the columns of sines is 0.80038, which is the sine of 53° 10′.

This last calculation would not have been necessary, because, as has already been mentioned, angle B could have been found directly when angle C was known, by the formula

$$B = 90° - C = 90° - 36° 50' = 53° 10'.$$

It will be noted that either method for finding angle B gives the same result.

As a further example, assume that one of the sides forming the right angle is 15 inches and the other is 36 inches, as shown in Fig. 2. Find the hypotenuse and the angles B and C.

The hypotenuse is found by Formula (1), previously given, the values being inserted.

$$\text{Hypotenuse} = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39.$$

The side opposite an angle divided by the side adjacent equals the tangent of the angle. Hence,

$$\tan B = \frac{36}{15} = 2.4.$$

By referring to the tables, it will be found that the nearest value to 2.4 in the columns of tangents is 2.3944, which is the tangent of 67° 20′. Hence,

$$B = 67° 20', \quad \text{and} \quad C = 90° - 67° 20' = 22° 40'.$$

Case 2. — If the hypotenuse and one acute angle are known, the side adjacent to the known angle is found by multiplying the hypotenuse by the cosine of the known angle; the side opposite the known angle is found by multiplying the hypotenuse by the sine of the known angle; and the other acute angle is found by subtracting the known angle from 90 degrees.

This rule may be expressed by simple formulas. Referring to Fig. 1, if a is the hypotenuse, and B the known angle, then:

$c = a \times \cos B$; $b = a \times \sin B$; $C = 90° - B$.

If C is the known angle, then:

$b = a \times \cos C$; $c = a \times \sin C$; $B = 90° - C$.

Example. — Assume that the hypotenuse $a = 22$ inches, and angle $B = 41$ degrees 40 minutes. Find sides b and c and angle C.

$c = a \times \cos B = 22 \times \cos 41° 40' = 22 \times 0.74702 = 16.434$ inches.
$b = a \times \sin B = 22 \times \sin 41° 40' = 22 \times 0.66479 = 14.625$ inches.
$C = 90° - 41° 40' = 48° 20'$

Case 3. — When one acute angle and its adjacent side are known, the hypotenuse is found by dividing the adjacent side by the cosine of the known angle; the side opposite the known angle is found by multiplying the known adjacent side by the tangent of the known angle; and the other acute angle is found by subtracting the known angle from 90 degrees.

Referring to Fig. 1, this rule can be expressed by simple formulas. If B is the known angle, and c the known side adjacent to angle B, then:

$a = \frac{c}{\cos B}$; $b = c \times \tan B$; $C = 90° - B$.

If C is the known angle, and b the known side, adjacent to angle C, then:

$a = \frac{b}{\cos C}$; $c = b \times \tan C$; $B = 90° - C$.

Case 4. — When one acute angle and the side opposite it are known, the hypotenuse is found by dividing the known side by the sine of the known angle; the side adjacent to the known angle is found by multiplying the known opposite side by the cotangent of the known angle; and the other acute angle is found by subtracting the known angle from 90 degrees.

If B is the known angle (see Fig. 1), and b the side opposite, which is also known, then:

$a = \frac{b}{\sin B}$; $c = b \times \cot B$; $C = 90° - B$.

If C is the known angle, and c the known side, opposite to angle C, then:

$$a = \frac{c}{\sin C}; \qquad b = c \times \cot C; \qquad B = 90^\circ - C.$$

Practical Problems requiring the Solution of Right-angled Triangles. — A great many of the problems encountered in the machine shop and tool-room require the solution of right-angled triangles, as will be apparent after studying the various practical examples found throughout this book. In fact, a general knowledge of the subject of trigonometry is of great

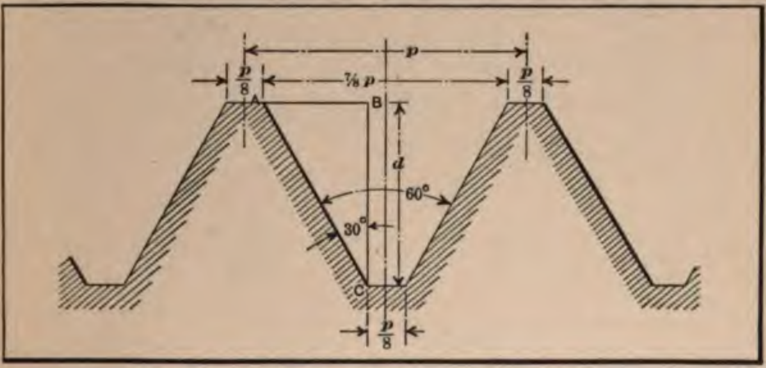


Fig. 3. Section of U. S. Standard Screw Thread

value to the machinist, toolmaker, and shop foreman as well as to the draftsman. Problems from practice will be given to illustrate just how the rules and formulas previously given are applied, and these will be supplemented by various other examples in the different chapters.

Example 1. — Fig. 3 shows a section of a U. S. standard thread. The problem is to find a formula for the depth of the thread for a given pitch, and calculate the depth of screw threads with 12 and 16 threads per inch.

In the illustration, p is the pitch of the thread. The pitch equals $\frac{1}{\text{No. of threads per inch}}$. It is required to find the depth BC of the thread, expressed in terms of the pitch. This depth can be found if we can solve the triangle ABC.

In the U. S. standard thread system there is a flat at the top and bottom of the thread as shown. The width of this flat is one-eighth of the pitch, as indicated. Hence, side *AB* of the right-angled triangle *ABC* equals one-half of $\frac{7}{8}$ pitch minus one-half of $\frac{1}{8}$ pitch, or $\left(\frac{7}{16} - \frac{1}{16}\right)$ pitch $= \frac{3}{8}$ pitch. The angle opposite this side is also known; it is one-half of the total thread angle, or 30 degrees. According to the rules and formulas

$$BC = AB \times \cot 30°.$$

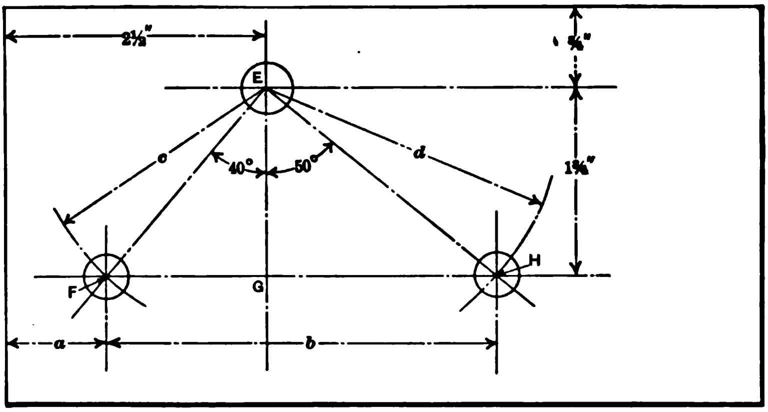


Fig. 4. Master Jig-plate

By inserting in this formula $BC = d$, $AB = \frac{3}{8} \cdot p$, and $\cot 30° = 1.7320$, we have:

$$d = \frac{3}{8}\, p \times 1.7320 = 0.6495\, p,$$

in which d = depth of thread; p = pitch of thread.

We will now find the depth of the thread for 12 and 16 threads per inch. As $p = \frac{1}{\text{No. of threads per inch}}$, we have, by inserting the known values in the general formula just found:

$$d = 0.6495 \times \frac{1}{12} = 0.0541 \text{ inch, for 12 threads;}$$

$$d = 0.6495 \times \frac{1}{16} = 0.0406 \text{ inch, for 16 threads.}$$

Example 2. — In laying out a master jig-plate, it is required that holes F and H, Fig. 4, shall be on a straight line which is $1\frac{3}{4}$ inch distant from hole E. The holes must also be on lines making, respectively, 40- and 50-degree angles with line EG, drawn at right angles to the sides of the jig-plate through E, as shown. Find the dimensions necessary for the toolmaker.

The dimensions which ought to be given the toolmaker in addition to those already given are indicated by a, b, c, and d. The two latter are the radii of the arcs which, if struck with E as a center, will pass through the centers of F and H. There

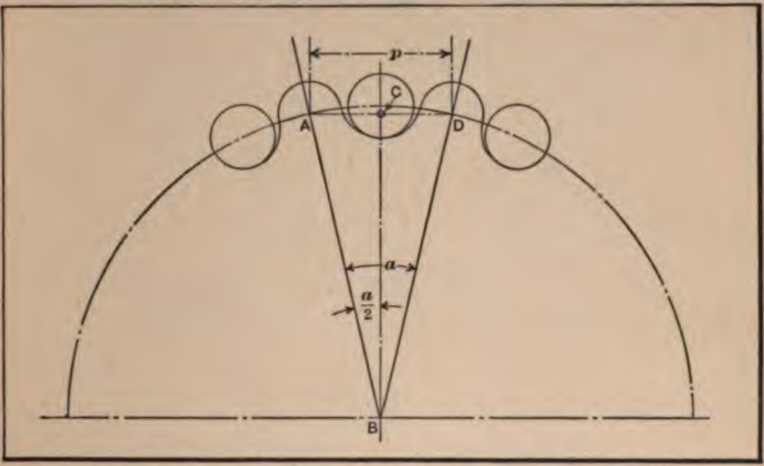


Fig. 5. Diagram of Sprocket Wheel for Roller Chain

are two right-angled triangles EFG and EGH. One acute angle in each is known, and also the length of side EG ($1\frac{3}{4}$ inch) which is the same for both triangles and is the side adjacent to the known angle.

$$FG = 1.75 \times \tan 40^\circ = 1.75 \times 0.83910 = 1.4684 \text{ inch;}$$

$$FE = \frac{1.75}{\cos 40^\circ} = \frac{1.75}{0.76604} = 2.2845 \text{ inches;}$$

$$GH = 1.75 \times \tan 50^\circ = 1.75 \times 1.1917 = 2.0856 \text{ inches;}$$

$$EH = \frac{1.75}{\cos 50^\circ} = \frac{1.75}{0.64279} = 2.7225 \text{ inches.}$$

By referring to Fig. 4 it will be seen that $FE = c$; $EH = d$; $2\frac{1}{2} - FG = a$; and $FG + GH = b$. Hence,

$$a = 2.5 - 1.4684 = 1.0316 \text{ inch;}$$
$$b = 1.4684 + 2.0856 = 3.5540 \text{ inches;}$$
$$c = 2.2845 \text{ inches;}$$
$$d = 2.7225 \text{ inches.}$$

Example 3. — If the pitch p of a roller chain is $\frac{3}{4}$ inch, and the sprocket wheel is to have 32 teeth, what will be the pitch diameter of the gear? (See Fig. 5.)

By referring to the engraving, it will be seen that $AD = p = \frac{3}{4}$ inch, and $AC = \frac{1}{2}\ AD = \frac{3}{8}$ inch, in this case. Line AB is the

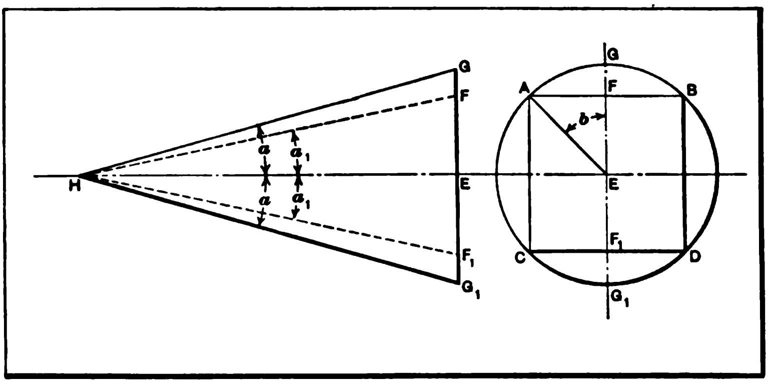


Fig. 6. Diagram of Flat-sided Taper Reamer

pitch radius or one-half the pitch diameter. Angle a is the angle for one tooth, and as the whole circle is 360 degrees, a in this case equals $\frac{360}{32} = 11\frac{1}{4}$ degrees, or 11 degrees 15 minutes. One-half of a, then, equals 5 degrees 37 minutes, approximately. We, therefore, have a right-angled triangle in which the length of side AC and the angle opposite to it are known, and it is necessary to find the hypotenuse AB.

$$AB = \frac{AC}{\sin \frac{a}{2}} = \frac{0.375}{\sin 5^\circ 37'} = \frac{0.375}{0.09787} = 3.832 \text{ inches.}$$

The pitch diameter, then, equals $2 \times 3.832 = 7.664$ inches.

Example 4. — Small reamers are sometimes provided with flats instead of actual flutes. The diameter of the reamer is, of course, measured over the sharp corners; if the reamer tapers, the taper of the flats will not be the same as the taper of the sharp corners, and the milling machine dividing-head must be set to a different angle from that which the cutting edge makes with the center line. A simple formula may be deduced by the aid of trigonometry for finding the angle to which to set the dividing-head when milling the flats.

Referring to Fig. 6, in which the reamer is imagined as continued to a sharp point at the end, let

a = angle made by cutting edge with center line;
a_1 = angle made by flat with center line;
N = number of sides of reamer;
T = taper per foot.

Angle b, can be determined by the formula:

$$b = \frac{360}{2N},$$

as is evident from the illustration.

Angle a_1 is the angle sought. It will be seen that if FE and HE were known, then

$$\text{Tan } a_1 = \frac{FE}{HE},$$

but $FE = AE \times \cos b$.

By inserting this value,

$$\text{Tan } a_1 = \frac{AE \times \cos b}{HE}.$$

As $\cos b = \cos \frac{360}{2N}$, then:

$$\text{Tan } a_1 = \frac{AE}{HE} \times \cos \frac{360}{2N}.$$

The distance AE, however, is one-half of the taper in the distance HE. The taper per inch, then, is $\frac{2AE}{HE}$, and the taper per foot,

$$T = 12 \times \frac{2AE}{HE} = \frac{24AE}{HE}, \text{ or } \frac{T}{24} = \frac{AE}{HE}.$$

If $\frac{T}{24}$ is inserted in the formula above, we have:

$$\text{Tan } a_1 = \frac{T}{24} \times \cos \frac{360}{2N}.$$

Assume that the taper per foot is $\frac{1}{4}$ inch, and that a four-sided reamer is required. Find the angle to which to set the index-head.

$$\text{Tan } a_1 = \frac{\frac{1}{4}}{24} \times \cos 45° = 0.00736,$$

which gives $a_1 = 25$ minutes.

Example 5. — In Fig. 7 are shown two pulleys of 6 and 12 inches diameter, with a fixed center distance of 5 feet. Find the length of belt required to pass over the two pulleys. The belt is assumed to be perfectly tight.

The length of the belt is made up of the two straight portions AC and BD, tangent to the circles as shown, and of the arc AEB of the larger pulley and the arc CFD of the smaller pulley. AC and BD are equal. First find the length AC. By drawing a line HG from H, the center of the smaller pulley, parallel to AC, we can construct a triangle HGK in which $HG = AC$, and $GK = AK - HC$. That $HG = AC$ is clear from the fact that HC and KA are parallel, both being perpendicular or at right angles to the tangent line AC. The figure $HGAC$ is, therefore, a rectangle, and hence, opposite sides are equal. Therefore, HG equals AC, and $HC = GA$.

That $GK = AK - HC$ is evident from the fact that $GK = AK - GA$, but as $GA = HC$, it follows that $GK = AK - HC$.

Now, AK is the radius of the larger pulley, which is one-half its diameter, or 6 inches, and HC is the radius of the smaller pulley or 3 inches. Hence, $GK = 6 - 3 = 3$ inches. $HK = 5$ feet or 60 inches, as given in the problem. Then this is a right-angled triangle in which the hypotenuse $HK = 60$ inches, and one of the sides forming the right angle is 3 inches. Hence, side GH is found by a previous formula given for this case, and by inserting the known values it reads:

$$GH = \sqrt{60^2 - 3^2} = \sqrt{3600 - 9} = \sqrt{3591} = 59.925.$$

As $GH = AC$, then $AC = 59.925$, and as $AC = BD$, we have $AC + BD = 119.85$ inches. It now remains to find the lengths of the circular arcs AEB and CFD. In order to find these lengths the number of degrees in these arcs must first be found and to find this, the first step is to find angle a. According to rules previously given:

$$\text{Cos } a = \frac{GK}{KH} = \frac{3}{60} = 0.05.$$

From this, it is found from the trigonometric tables that $a = 87° 8'$. Angle $AKE = 180° - a = 180° - 87° 8' = 92° 52'$.

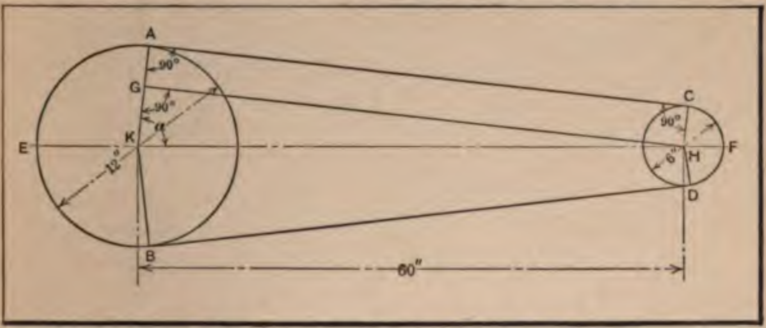


Fig. 7. Diagram of Belt and Pulley Drive

Angle EKB = angle AKE, so that the arc AEB, therefore, is equal to twice angle AKE, or:

$$\text{Arc } AEB = 2 \times 92° 52' = 185° 44'.$$

The whole circumference of the larger pulley equals $3.1416 \times 12 = 37.699$ inches. As the whole circumference is 360 degrees, its length in inches is to the length of arc AEB as 360 degrees is to $185° 44'$, or:

$$\frac{37.699}{\text{arc } AEB} = \frac{360°}{185° 44'}.$$

Transposing this expression:

$$\text{Arc } AEB = \frac{37.699 \times 185° 44'}{360°}.$$

Before this calculation can be carried out, 44 minutes must be transformed to decimals of a degree. As 44 minutes equals

$\frac{44}{60}$ of a degree, this, changed to a decimal fraction, equals $\frac{44}{60} = 0.73$, and 185° 44′ equals 185.73 degrees. Then:

$$\text{Arc } AEB = \frac{37.699 \times 185.73}{360} = 19.45 \text{ inches.}$$

Now, to find arc *CFD*, angle *CHF* is first determined. This angle equals angle *GKH* or *a*, because *AK* and *CH* are parallel lines. Hence, arc $CFD = 2 \times \text{angle } a = 2 \times 87° \, 8' = 174° \, 16'$. Now, proceeding as before:

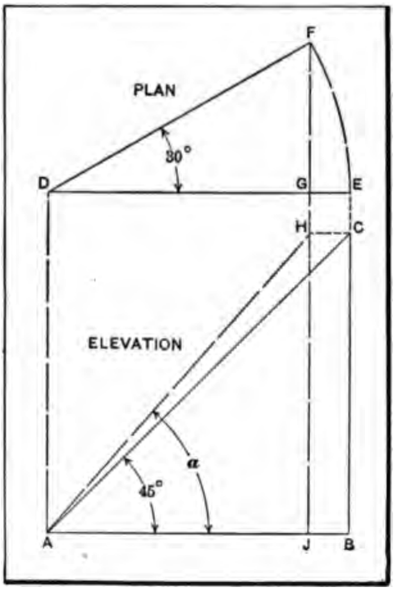


Fig. 8. Diagram illustrating Solution of Double or Compound Angles

$3.1416 \times 6 = 18.8496 =$ circumference of small pulley.

$$\frac{18.8496}{\text{arc } CFD} = \frac{360°}{174° \, 16'}$$

Transposing this and changing 16 minutes to decimals of a degree:

Arc *CFD* =

$$\frac{18.8496 \times 174.27}{360} = 9.12 \text{ in.}$$

The total length of the belt, then, equals:

$$119.85 + 19.45 + 9.12 = 148.42 \text{ inches.}$$

Double or Compound Angles. — Many men in the shop and tool-room are familiar with the formulas employed in solving problems in which right-angled triangles are involved, and a great many can also work out problems requiring the solution of oblique-angled triangles, but the method of solving double or compound angles is not so well known. The following, however, is a very simple method: Suppose that a 45-degree angle on elevation *CAB* (see Fig. 8) is to be swung 30 degrees in a horizontal plane about point *A*. The plan view of this angle before being swung around would be a straight line DE equal to the length of the base AB. Now swing the line DE through an arc of 30 degrees to the position DF and draw a line perpendicular to line AB from the point F. Call this line FJ and lay off HJ equal to CB. Triangle HAJ is now the true elevation of the triangle in its new position DF, the side AC being represented by line AH, and side BC, by side JH. Now:

$$AB = DE = DF, \qquad \text{and} \qquad BC = HJ.$$

$BC = AB$ tan 45 degrees; therefore, $HJ = AB$ tan 45 degrees.

Further, $DG = DF$ cos 30 degrees; and as $DG = AJ$, $AJ = DF$ cos 30 degrees $= AB$ cos 30 degrees. Now:

$$\text{Tan } a = \frac{HJ}{AJ},$$

or,

$$\text{Tan } a = \frac{AB \text{ tan 45 deg.}}{AB \text{ cos 30 deg.}} = \frac{\text{tan 45 deg.}}{\text{cos 30 deg.}}$$

This principle can be carried a step further and a compound angle worked out in the same manner, resolving the problem into a series of simple motions. In using this method any convenient side of the triangle may be taken as unity and the problem solved with the trigonometric functions of the angles.

CHAPTER XII

SOLUTION OF OBLIQUE-ANGLED TRIANGLES

The methods used in the solution of oblique triangles — that is, triangles in which none of the angles is a right angle — differ according to which parts are known and which are to be found. The problems which present themselves may be divided into four classes:

1. Two angles and one side known.
2. Two sides and the angle included between them known.
3. Two sides and the angle opposite one of them known.
4. The three sides known.

Two Angles and One Side known. — Assume that the angles A and B in Fig. 1 are given as shown, and that side a is 5 inches. Find angle C, and sides b and c.

As the sum of the three angles in a triangle always equals 180 degrees, angle C can be found directly when angles A and B are given, by subtracting the sum of these angles from 180 degrees. Angle A = 80 degrees and B = 62 degrees; therefore,

$$C = 180° - (80° + 62°) = 180° - 142° = 38°.$$

For finding the sides b and c the following rule is used: The side to be found equals the known side multiplied by the sine of the angle opposite the side to be found, and the product divided by the sine of the angle opposite the known side.

To find side b, for example, multiply the known side a by the sine of angle B, and divide the product by the sine of angle A. Written as a formula this would be:

$$b = \frac{a \times \sin B}{\sin A}. \qquad (1)$$

In the same way

$$c = \frac{a \times \sin C}{\sin A}. \qquad (2)$$

If the known values are inserted for side a and the angles in these formulas, they read:

$$b = \frac{5 \times \sin 62^\circ}{\sin 80^\circ} = \frac{5 \times 0.88295}{0.98481} = 4.483 \text{ inches;}$$

$$c = \frac{5 \times \sin 38^\circ}{\sin 80^\circ} = \frac{5 \times 0.61566}{0.98481} = 3.126 \text{ inches.}$$

Example 1. — In Fig. 2 is shown a triangle of which one side is 6.5 feet, and the two angles A and C (78 and 73 degrees, respectively) are given. Find angle B and sides b and c.

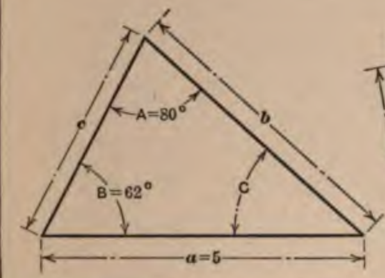


Fig. 1. Two Angles and One Side Known

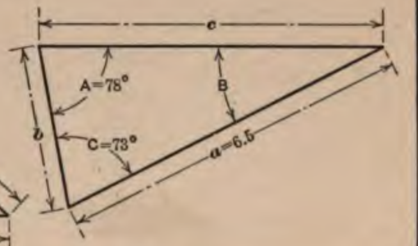


Fig. 2. Example 1

Using the same method as explained for finding angle C in the previous example:

$$B = 180^\circ - (78^\circ + 73^\circ) = 180^\circ - 151^\circ = 29^\circ.$$

For finding sides b and c, use the rule or formulas previously given, inserting the values given in this example:

$$b = \frac{a \times \sin B}{\sin A} = \frac{6.5 \times \sin 29^\circ}{\sin 78^\circ} = \frac{6.5 \times 0.48481}{0.97815}$$

$$= \frac{3.151265}{0.97815} = 3.222 \text{ feet;}$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{6.5 \times \sin 73^\circ}{\sin 78^\circ} = \frac{6.5 \times 0.95630}{0.97815}$$

$$= \frac{6.21595}{0.97815} = 6.355 \text{ feet.}$$

Example 2. — In Fig. 3, side a equals 3.2 inches, angle A, 118 degrees, and angle B, 40 degrees. Find angle C and sides b and c.

$$C = 180^\circ - (118^\circ + 40^\circ) = 180^\circ - 158^\circ = 22^\circ;$$

$$b = \frac{3.2 \times \sin 40^\circ}{\sin 118^\circ} = \frac{3.2 \times 0.64279}{0.88295} = 2.330 \text{ inches.}$$

Note, when finding sin 118° from the tables, that $\sin 118^\circ = \sin (180^\circ - 118^\circ) = \sin 62^\circ$.

$$c = \frac{3.2 \times \sin 22^\circ}{\sin 118^\circ} = \frac{3.2 \times 0.37461}{0.88295} = 1.358 \text{ inch.}$$

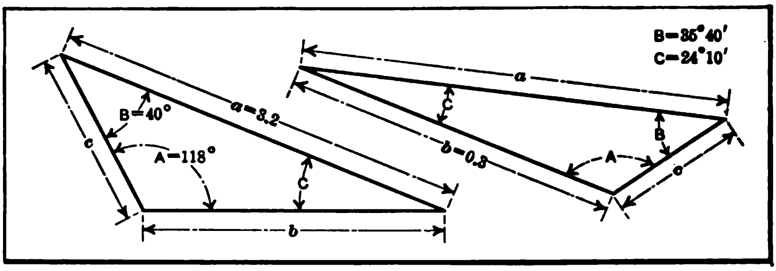


Fig. 3. Example 2 Fig. 4. Example 3

Example 3. — In Fig. 4, side $b = 0.3$ foot, angle $B = 35^\circ\ 40'$, and angle $C = 24^\circ\ 10'$. Find angle A and sides a and c.

$$A = 180^\circ - (35^\circ\ 40' + 24^\circ\ 10') = 180^\circ - 59^\circ\ 50' = 120^\circ\ 10';$$

$$a = \frac{b \times \sin A}{\sin B} = \frac{0.3 \times \sin 120^\circ\ 10'}{\sin 35^\circ\ 40'} = \frac{0.3 \times 0.86457}{0.58307} = 0.445 \text{ foot;}$$

$$= \frac{b \times \sin C}{\sin B} = \frac{0.3 \times \sin 24^\circ\ 10'}{\sin 35^\circ\ 40'} = \frac{0.3 \times 0.40939}{0.58307} = 0.211 \text{ foot.}$$

Note that in this example the formulas for a and c have the same *form* as Formulas (1) and (2), but as the side b is the known side, instead of a, the side b is brought into the formula instead of a, and angle B instead of angle A.

Summary of Formulas: If the angles of a triangle are called A, B, and C, and the sides opposite each of the angles, a, b, *and* c, respectively, as shown in Fig. 1, then, if two angles and one side are known, the two unknown sides may be found by the formulas below:

$$a = \frac{b \times \sin A}{\sin B}; \qquad b = \frac{a \times \sin B}{\sin A}; \qquad c = \frac{b \times \sin C}{\sin B};$$

$$a = \frac{c \times \sin A}{\sin C}; \qquad b = \frac{c \times \sin B}{\sin C}; \qquad c = \frac{a \times \sin C}{\sin A}.$$

Two Sides and the Included Angle known. — Assume that the sides a and b in Fig. 5 are 9 and 8 inches, respectively, as shown, and that the angle C formed by these two sides is 35 degrees. Find angles A and B, and side c, of the triangle.

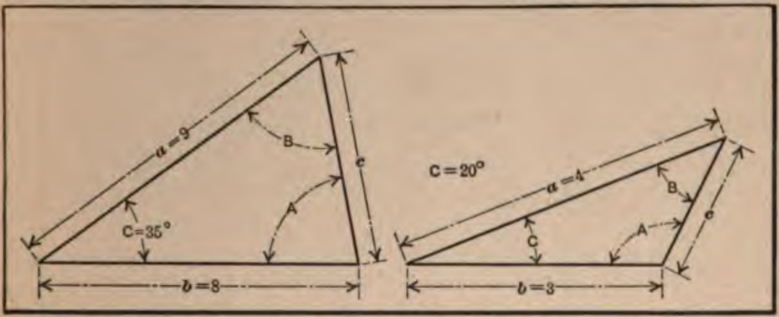


Fig. 5. Two Sides and One Angle Known **Fig. 6. Example 1**

The tangent of angle A is found by the following formula:

$$\text{Tan } A = \frac{a \times \sin C}{b - a \times \cos C}. \qquad (3)$$

If the given values of a, b, and C are inserted in this formula, it reads:

$$\text{Tan } A = \frac{9 \times \sin 35^\circ}{8 - 9 \times \cos 35^\circ} = \frac{9 \times 0.57358}{8 - 9 \times 0.81915}$$

$$= \frac{5.16222}{0.62765} = 8.22468.$$

The tangent of angle $A = 8.22468$, having now been obtained, it is found from tables that the angle equals $83^\circ\ 4'$.

Both angles A and C being known,

$$\text{Angle } B = 180^\circ - (A + C) = 180^\circ - (83^\circ\ 4' + 35^\circ)$$

$$= 180^\circ - 118^\circ\ 4' = 61^\circ\ 56'.$$

Side c is found by Formula (2):

$$c = \frac{a \times \sin C}{\sin A} = \frac{9 \times \sin 35^\circ}{\sin 83^\circ 4'} = \frac{9 \times 0.57358}{0.99269} = 5.2 \text{ inches.}$$

All the required quantities of this triangle have now been found.

Example 1. — In Fig. 6, $a = 4$ inches, $b = 3$ inches, and $C = 20$ degrees. Find A, B, c, and the area.

According to Formula (3):

$$\text{Tan } A = \frac{a \times \sin C}{b - a \times \cos C} = \frac{4 \times \sin 20^\circ}{3 - 4 \times \cos 20^\circ} = \frac{4 \times 0.34202}{3 - 4 \times 0.93969}$$
$$= \frac{1.36808}{3 - 3.75876}.$$

It will be seen that in the denominator of the fraction above, the number to be subtracted from 3 is greater than 3; the numbers are, therefore, reversed, 3 being subtracted from 3.75876, the remainder then being negative. Hence:

$$\text{Tan } A = \frac{1.36808}{3 - 3.75876} = \frac{1.36808}{-0.75876} = -1.80305.$$

The final result is negative because a positive number (1.36808) is divided by a negative number (−0.75876). The tangents of angles greater than 90 degrees and smaller than 180 degrees are negative. Find in this case the value nearest to 1.80305 in the columns of tangents in the tables. In a table containing values for each minute, the nearest value is 1.8028, which is the tangent of 60° 59′. As the tangent is negative, angle A is not 60° 59′, but equals 180° − 60° 59′ = 119° 1′.

Now angle B is found by the formula:

$$B = 180^\circ - (A + C) = 180^\circ - (119^\circ 1' + 20^\circ)$$
$$= 180^\circ - 139^\circ 1' = 40^\circ 59'.$$

Side c is now found by the same formulas and in the same manner as previously shown.

Example 2. — In Fig. 7, $a = 7$ feet, $b = 4$ feet, and $C = 121$ degrees. Find A, B, and c.

As previously explained:

$$\sin 121^\circ = \sin(180^\circ - 121^\circ) = \sin 59^\circ, \quad \text{and}$$
$$\cos 121^\circ = -\cos(180^\circ - 121^\circ) = -\cos 59^\circ.$$

Therefore,

$$\text{Tan } A = \frac{7 \times \sin 121^\circ}{4 - 7 \times \cos 121^\circ} = \frac{7 \times \sin 59^\circ}{4 - 7 \times (-\cos 59^\circ)}$$
$$= \frac{7 \times 0.85717}{4 - 7 \times (-0.51504)} = \frac{6.00019}{4 - (-3.60528)}$$
$$= \frac{6.00019}{4 + 3.60528} = \frac{6.00019}{7.60528} = 0.78895.$$

The calculation with the negative number (−0.51504) will become clear by comparing the foregoing processes with the

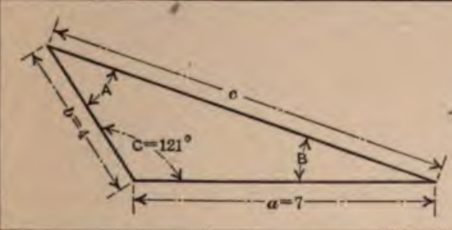


Fig. 7. Example 2

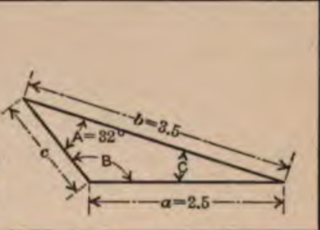


Fig. 8. Sides *a* and *b* and Angle *A* known

rules given in Chapter II (see "Positive and Negative Quantities"). When multiplied by 7, the product $7 \times (-0.51504)$ becomes negative, and equals −3.60528. As subtracting a negative quantity from a positive quantity is equal to adding the numerical value of the negative number, then:

$$4 - (-3.60528) = 4 + 3.60528 = 7.60528.$$

Having found tan $A = 0.78895$, we find angle A from the tables: $A = 38^\circ\ 16'$.

Angle B and side c are now found in the same way as previously explained.

Summary of Formulas: If the angles of a triangle are called A, B, and C and the sides opposite each of the angles, a, b, c, respectively, as shown in Fig. 5, then, if any two sides and the included angle are known, the other angles and the remaining side may be found. One of the angles is first found by any of the following formulas:

$$\text{Tan } A = \frac{a \times \sin C}{b - a \times \cos C}; \qquad \tan A = \frac{a \times \sin B}{c - a \times \cos B};$$

$$\text{Tan } B = \frac{b \times \sin C}{a - b \times \cos C}; \qquad \tan B = \frac{b \times \sin A}{c - b \times \cos A};$$

$$\text{Tan } C = \frac{c \times \sin B}{a - c \times \cos B}; \qquad \tan C = \frac{c \times \sin A}{b - c \times \cos A}.$$

The remaining side is found by using Formulas (1) and (2), and the third angle by subtracting the sum of the known angles from 180 degrees as previously explained.

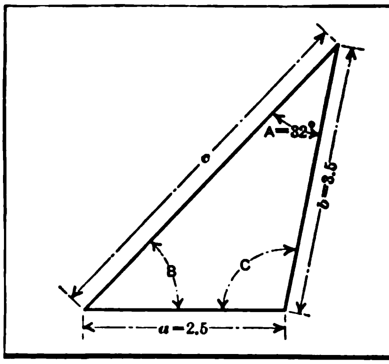


Fig. 9. Sides a and b and Angle A similar to Fig. 8

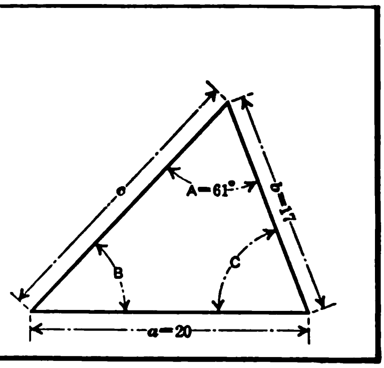


Fig. 10. Example 1

If the unknown angles are not required, but merely the unknown side of the triangle, the following formulas may be employed:

$$a = \sqrt{b^2 + c^2 - 2\,bc \times \cos A};$$

$$b = \sqrt{a^2 + c^2 - 2\,ac \times \cos B};$$

$$c = \sqrt{a^2 + b^2 - 2\,ab \times \cos C}.$$

Two Sides and One of the Opposite Angles known. — When two sides and the angle opposite one of the given sides are known, two triangles can be drawn which have the sides the required length, and the angle opposite one of the sides, the required size. In Fig. 8 is shown a triangle in which side a is 2.5 inches, side b, 3.5 inches, and angle A, 32 degrees.

Another triangle is shown in Fig. 9 in which sides a and b have the same length as the triangle just referred to, and angle A opposite side a still remains 32 degrees; but it will be seen that in this triangle the angle B is very much smaller than in the triangle shown in Fig. 8. In every case, therefore, when two sides and one of the opposite angles are given, the problem is capable of two solutions, there being two triangles which fill the given requirements. In one of these triangles, the unknown angle opposite a given side is greater than a right angle, and in one it is less than a right angle. When the triangle to be calculated is drawn to the correct shape, it is, therefore, possible to determine from the shape of the triangle which of the two solutions applies. When the triangle is not drawn to the required shape, both solutions must be found and applied to the practical problem requiring the solution of the triangle; it can then usually be determined which of the solutions applies to the practical problem in hand.

Example 1. — Assume that the sides a and b in Fig. 10 are 20 and 17 inches, respectively, as shown, and that angle A opposite the known side a is 61 degrees. Find angles B and C and side c of the triangle.

The angle B opposite the known side b may be found by the following rule:

Rule: The sine of the angle opposite one of the known sides equals the product of the side opposite this angle times the sine of the known angle, divided by the side opposite the known angle.

From this rule the following formula for the sine of angle B is derived:

$$\text{Sin } B = \frac{b \times \sin A}{a}. \quad (4)$$

If the known values for sides b and a and angle A are inserted in this formula, then:

$$\text{Sin } B = \frac{17 \times \sin 61^\circ}{20} = \frac{17 \times 0.87462}{20} = 0.74343.$$

Having sin $B = 0.74343$, it is found from the tables that $B = 48° 1'$. As it is shown in Fig. 10 that angle B is less than a right angle, the solution found is the one which applies in this case.

Angle $C = 180° - (A + B) = 180° - (61° + 48° 1') = 70° 59'$.

Side c is found by Formula (2):

$$c = \frac{a \times \sin C}{\sin A} = \frac{20 \times \sin 70° 59'}{\sin 61°} = \frac{20 \times 0.94542}{0.87462} = 21.62 \text{ inches.}$$

Example 2. — In Fig. 11, $a = 5$ inches, $b = 7$ inches, and $A = 35$ degrees. Find B, C, and c.

According to the rule and formula in the previous example:

$$\text{Sin } B = \frac{b \times \sin A}{a} = \frac{7 \times \sin 35°}{5} = \frac{7 \times 0.57358}{5} = 0.80301.$$

Having sin $B = 0.80301$, it is found from the tables that $B = 53° 25'$. However, in the present case it is seen from the figure that B is greater than 90 degrees. The solution obtained is, therefore, not the solution applying to this case. The sine of an angle also equals the sine of 180 degrees minus the angle. Therefore, 0.80301 is the sine not only of $53° 25'$, but also of $180° - 53° 25' = 126° 35'$. The value of angle B applying to the triangle shown in Fig. 11 is, therefore, $126° 35'$, because of the two values obtained, this is the one which is greater than a right angle.

Example 3. — In Fig. 12, $a = 2$ feet, $b = 3$ feet, and $A = 30$ degrees. Find B, C, and c.

The sine of angle B is found as in the previous example:

$$\text{Sin } B = \frac{b \times \sin A}{a} = \frac{3 \times \sin 30°}{2} = 0.75000.$$

Having sin $B = 0.75000$, it is found from the tables that $B = 48° 35'$. From Fig. 12, it is apparent, however, that B is greater than 90 degrees, and as 0.75000 is the sine not only of $48° 35'$, but also of $180° - 48° 35' = 131° 25'$, angle B in this case equals $131° 25'$.

When the angle B is found, angle C and side c are found in the same manner as in Example 1.

Summary of Formulas: If the angles of a triangle are called A, B, and C, and the sides opposite each of the angles, a, b, and c, respectively, as shown in Fig. 9, then if any two sides and one angle opposite one of the known sides are given, the other angles and the remaining side may be found. The angle opposite the other known side is first found by any of the formulas below:

$$\text{Sin } A = \frac{a \times \sin B}{b}; \qquad \sin A = \frac{a \times \sin C}{c};$$

$$\text{Sin } B = \frac{b \times \sin A}{a}; \qquad \sin B = \frac{b \times \sin C}{c};$$

$$\text{Sin } C = \frac{c \times \sin A}{a}; \qquad \sin C = \frac{c \times \sin B}{b}.$$

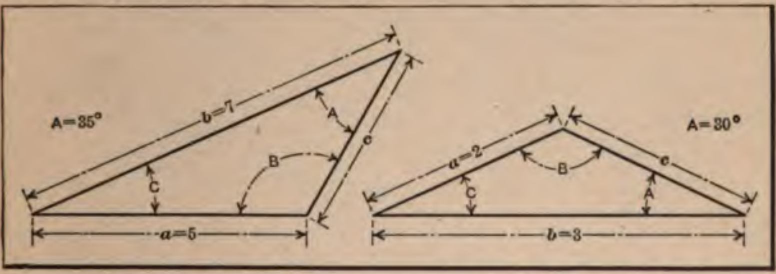


Fig. 11. Example 2

Fig. 12. Example 3

The remaining side is found by Formulas (1) or (2), and the third angle by subtracting the sum of the known angles from 180 degrees.

Three Known Sides. — *Example* 1. — In Fig. 13 the three sides a, b, and c of the triangle are given; $a = 8$ inches, $b = 9$ inches, and $c = 10$ inches. Find the angles A, B, and C.

Any of the angles can be found by the formulas given below:

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}; \tag{5}$$

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2 \times a \times c}; \tag{6}$$

$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2 \times a \times b}. \tag{7}$$

If the given lengths of the sides are inserted in the first of the formulas above, then:

$$\text{Cos } A = \frac{9^2 + 10^2 - 8^2}{2 \times 9 \times 10} = \frac{81 + 100 - 64}{180} = \frac{117}{180} = 0.65000.$$

Having cos A = 0.65000, it is found from the tables that angle $A = 49° \ 27'$.

Having found angle A the easiest method for finding angle B is by Formula (4). From this formula:

$$\text{Sin } B = \frac{b \times \sin A}{a} = \frac{9 \times \sin 49° \ 27'}{8} = \frac{9 \times 0.75984}{8} = 0.85482.$$

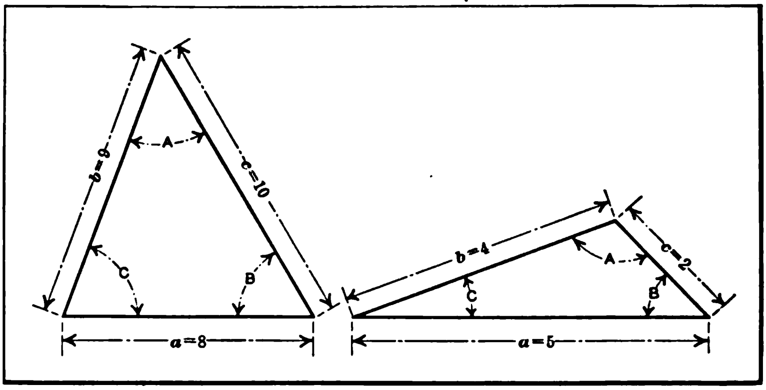


Fig. 13. Three Known Sides — Example 1

Fig. 14 Three Known Sides— Example 2

Having sin B = 0.85482, it is found from the tables that $B = 58° \ 44'$.

Angle $C = 180° - (A + B) = 180° - (49° \ 27' + 58° \ 44') = 71° \ 49'$.

Example 2. — In Fig. 14, a = 5 inches, b = 4 inches, and c = 2 inches. Find the angles of the triangle.

Using Formula (5):

$$\text{Cos } A = \frac{4^2 + 2^2 - 5^2}{2 \times 4 \times 2} = \frac{16 + 4 - 25}{16} = \frac{20 - 25}{16}.$$

It will be seen that in the numerator of the last fraction, *the number* to be subtracted from 20 is greater than 20. The numbers are, therefore, reversed, 20 being subtracted from 25, the remainder then being negative. Hence:

$$\text{Cos } A = \frac{20 - 25}{16} = \frac{-5}{16} = -0.31250.$$

The final result is negative, because a negative number (−5) is divided by a positive number (16). The cosines of angles greater than 90 degrees and smaller than 180 degrees are negative. Find the value nearest to 0.31250 in the columns of cosines in the tables. It will be seen that the nearest value is 0.31261, which is the cosine of 71° 47′. As the cosine here is negative, angle A, however, is not 71° 47′ but equals 180° − 71° 47′ = 108° 13′. Now angle B is found by the formula:

$$\text{Sin } B = \frac{b \times \sin A}{a} = \frac{4 \times \sin 108^\circ\ 13'}{5}.$$

Sin 108° 13′ = sin (180° − 108° 13′) = sin 71° 47′. Hence:

$$\text{Sin } B = \frac{4 \times \sin 71^\circ\ 47'}{5} = \frac{4 \times 0.94988}{5} = 0.75990, \text{ and}$$

$B = 49^\circ\ 27'$.

Finally, angle C is found by the formula:

$$C = 180^\circ - (A + B) = 180^\circ - (108^\circ\ 13' + 49^\circ\ 27') = 22^\circ\ 20'.$$

Areas of Triangles. — The area of any triangle equals one-half the product of the base and the altitude, as explained in Chapter IV. The length of the base and the altitude, however, are not always known and other values must be used in determining the area. For example, the lengths of the three sides may be known, or the lengths of two sides and the angle between them.

When Three Sides are given. — When only the lengths of the three sides of a triangle are given, the area is found as follows: Find one-half the sum of the three sides and subtract this half sum from each side; then find the product of these three remainders and multiply this product by the half sum. The square root of this final product equals the area. If the lengths of the sides are represented by A, B, and C, respectively, and D equals one-half the sum of the sides, then:

$$D = \frac{A + B + C}{2}.$$

$$\text{Area} = \sqrt{(D - A) \times (D - B) \times (D - C) \times D.}$$

Example. — What is the area of a triangle the sides of which are 6, 8, and 10 inches long, respectively?

$$\text{One-half the sum of the sides} = \frac{6 + 8 + 10}{2} = 12.$$

$$\text{Area} = \sqrt{(12 - 6) \times (12 - 8) \times (12 - 10) \times 12}$$
$$= \sqrt{6 \times 4 \times 2 \times 12} = \sqrt{576} = 24 \text{ square inches.}$$

When Two Sides and an Angle are given. — When the lengths of two sides of a triangle and the angle between the sides are given, the area may be found by multiplying one-half the product of the two sides by the sine of the angle between them.

In the example in Fig. 1, the area, then, equals one-half the product of sides a and b multiplied by the sine of angle C, or, expressed as a formula:

$$\text{Area} = \frac{a \times b \times \sin C}{2}.$$

If the known values for a and b are 5 and 4.483 inches, respectively, and angle C is 38 degrees, then:

$$\text{Area} = \frac{5 \times 4.483 \times \sin 38^\circ}{2} = \frac{5 \times 4.483 \times 0.61566}{2}$$
$$= \frac{13.8000}{2} = 6.9 \text{ square inches.}$$

The three values given in Fig. 5 are 8 and 9 inches for the sides and 35 degrees for the angle between the sides. Hence:

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{9 \times 8 \times 0.57358}{2} = 20.649 \text{ square inches.}$$

CHAPTER XIII

MILLING MACHINE INDEXING

THE index-head or dividing-head of a milling machine is constructed with a worm and worm-wheel mechanism, the worm being rotated by the crank when indexing, and the worm-wheel being mounted on the index-spindle to which the work is attached. By moving the crank with its index-pin a certain number of holes in one of the index circles, a certain angular movement can be imparted to the work. The calculating of indexing movements for the milling machine consists in finding how much the index-crank requires to be turned in order to produce the required movement for indexing whatever part is held by the work spindle.

Most of the regularly manufactured index-heads use a single-threaded worm engaging with a worm-wheel having 40 teeth. Thus, when the index-crank is turned around one full revolution, the worm is also revolved one complete turn, and this moves the worm-wheel one tooth, or $\frac{1}{40}$ of its circumference. Therefore, in order to turn the worm-wheel and the index-spindle on which it is mounted one full revolution, it is necessary to turn the index-crank 40 revolutions. If it is desired to revolve the index-spindle one-half revolution, the index-crank would have to be turned 20 revolutions. If it is desired to turn the index-spindle only one-fourth of a revolution, the index-crank is turned 10 revolutions.

Calculating the Indexing Movement. — Suppose that it is desired to mill the hexagonal head of a bolt. As the head has six sides, it is necessary to index it $\frac{1}{6}$ revolution. As it requires 40 revolutions of the index-crank to revolve the index-spindle once, it evidently requires only $\frac{1}{6}$ of that number to turn the index-spindle $\frac{1}{6}$ revolution; this is the amount that the work should be turned around or indexed when one side of the hexagon has been milled, and the next is ready to be milled. Consequently, the index-crank should be turned around $\frac{40}{6} = 6\frac{2}{3}$ revolutions for milling a hexagon; that is, the crank is first turned 6 full revolutions, and then, by means of the index-plate, it is turned two-thirds of a revolution. If the circle in the index-plate having 18 holes is used, two-thirds of a revolution will mean 12 holes in this circle, as 12 is two-thirds of 18 ($12 = \frac{2}{3} \times 18$).

Assume that a piece of work has eight sides regularly spaced (regular octagon). The indexing for each side is found by dividing 40 by 8. Thus $\frac{40}{8} = 5$, represents the number of revolutions of the index-crank for each side indexed and milled.

Assume that it is required to cut nine flutes regularly spaced in a reamer. The index-crank must be turned $\frac{40}{9} = 4\frac{4}{9}$ revolutions in order to index for each flute. The $\frac{4}{9}$ of a revolution would correspond to eight holes in the 18-hole circle, because $\frac{8}{18} = \frac{4}{9}$.

Assume that it is required to cut 85 teeth in a spur gear. The index-crank must be revolved $\frac{40}{85} = \frac{8}{17}$ revolutions to index for each tooth. To move the index-crank $\frac{8}{17}$ of a revolution corresponds to moving it 8 holes in the 17-hole circle.

General Rule for Indexing. — As a general rule for finding the number of revolutions required for indexing for any regular spacing, with any index-head, the following rule may be used:

Rule: To find the number of revolutions of the index-crank for indexing, divide the number of turns required of the index-crank for one revolution of the index-head spindle by the number of divisions required in the work.

[Most standard index-heads are provided with an index-plate fastened directly to the index-spindle for rapid direct indexing. This index-plate is usually provided with 24 holes, so that 2, 3, 4, 6, 8, 12, and 24 divisions can be obtained directly by the use of this direct index-plate without using the regular *indexing* mechanism. When using this index-plate for rapid direct indexing, no calculations are required, as the number of divisions obtainable by the use of the different holes in this plate are, as a rule, marked directly at the respective holes.]

Finding the Index Circle to Use. — In order to find which index circle to use and how many holes in that index circle to move for a certain fractional turn of the index-crank, the numerator and denominator of the fraction expressing the fractional turn are multiplied by the same number until the denominator of the new fraction equals the number of holes in some one index circle. The number with which to multiply must be found by trial. The numerator of the new fraction then expresses how many holes the crank is to be moved in the circle expressed by the denominator.

Assume that 12 flutes are to be drilled in a large tap. Assume that 40 turns of the index-crank are required for one turn of the index-head spindle. First divide 40 by 12 to find the number of turns of the index-crank required for each indexing. Thus:

$$\frac{40}{12} = 3\tfrac{4}{12} = 3\tfrac{1}{3}.$$

The fractional turn required is one-third of a revolution. Now multiply, according to the rule given, the numerator and denominator of this fraction by a number so selected that the new denominator equals the number of holes in some one index circle. Multiplying by 6 would give the following result:

$$\frac{1 \times 6}{3 \times 6} = \frac{6}{18},$$

in which the denominator 18 represents the number of holes in the index circle to use, and 6 is the number of holes the crank must be moved in this circle to turn the worm-shaft and worm one-third of a revolution.

Many milling machines are furnished with three index-plates, each having six index circles. The following numbers of holes in the index circles of the three index-plates are commonly used:

15	16	17	18	19	20
21	23	27	29	31	33
37	39	41	43	47	49

Indexing for Angles. — While most indexing is for a given number of divisions, it is sometimes necessary to rotate the work through a given angle by means of the dividing-head. In Fig. 1 is shown a piece of round stock having two flats milled in such a way that the angle between two lines from the center at right angles to the two surfaces is 35 degrees. In this case the index-head cannot be turned so as to make a certain whole number of moves in one complete revolution of the work, as is done, for instance, when four moves are made in one revolution for milling a square, six moves in one revolution for milling a hexagon, and 80 moves for milling an 80-tooth gear. Instead, here is given a certain number of degrees which it is required that the work be turned before another cut is taken by the milling cutter.

Indexing for angles is required only when an angle is given which is not such a simple fraction of the whole circle as, for instance, 90 degrees, which is $\frac{1}{4}$ of a complete turn, or 45 degrees, which is $\frac{1}{8}$ of a complete turn, or 60 degrees, which is $\frac{1}{6}$ of a complete turn; the numbers of turns of the index-crank in these cases are determined as previously explained. But if it be required to index for, say, 19 degrees, the method used is the one explained in the following.

Calculating the Movements for Angular Indexing. — There are 360 degrees in one complete circle or turn, and assuming that 40 turns of the index-crank are required for one revolution of the work, one turn of the index-crank must equal $\frac{360}{40} = 9$ degrees. Then, when one complete turn of the index-crank equals 9 degrees, two holes in the 18-hole circle, or 3 holes in the 27-hole circle, must correspond to one degree.

$$\frac{3}{27} = \frac{2}{18} = \frac{1}{9}.$$

The first principle or rule for indexing for angles is, therefore, that two holes in the 18-hole circle or 3 holes in the 27-hole circle equals a movement of one degree of the index-head spindle and the work.

Assume that an indexing movement of 35 degrees is required as shown in Fig. 1. One complete turn of the index-crank equals 9 degrees; therefore, first divide the number of degrees for which it is desired to index, by 9, in order to find how many complete turns the index-crank should make. The number of degrees left to turn when the full turns have been completed are indexed by taking two holes in the 18-hole circle for each degree. The indexing movement for 35 degrees

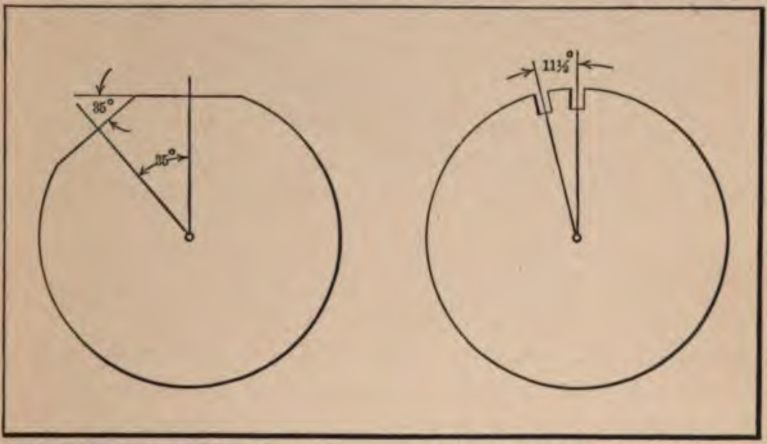


Fig. 1. Example of Work requiring Angular Indexing **Fig. 2. Another Example of Angular Indexing**

equals $\frac{35}{9} = 3\frac{8}{9}$, which indicates that the index-crank must be turned three revolutions, and then 8 degrees more must be indexed for, or 16 holes moved in the 18-hole circle.

Indexing a Fractional Part of a Degree. — Assume that it is desired to index $11\frac{1}{2}$ degrees, as shown in Fig. 2. Two holes in the 18-hole circle represent one degree, and consequently one hole represents $\frac{1}{2}$ degree. To index for $11\frac{1}{2}$ degrees, first turn the index-crank one revolution, this being a 9-degree movement. Then to index $2\frac{1}{2}$ degrees, the index-crank must be moved 5 holes in the 18-hole circle (4 holes for the two whole degrees and one hole for the $\frac{1}{2}$ degree equals the total movement of 5 holes).

Below is shown how this calculation may be carried out to indicate plainly the motion required for this angle:

$11\frac{1}{2}$ deg. = 9 deg. + 2 deg. + $\frac{1}{2}$ deg.
1 turn + 4 holes + 1 hole in the 18-hole circle.

Should it be required to index only $\frac{1}{3}$ degree, this may be made by using the 27-hole circle. In this circle a three-hole movement equals one degree, and a one-hole movement in the circle thus equals $\frac{1}{3}$ degree, or 20 minutes. Assume that it is required to index the work through an angle of 48 degrees 40 minutes. First turn the crank 5 turns for 45 degrees ($5 \times 9 = 45$). Then there are 3 degrees 40 minutes or $3\frac{2}{3}$ degrees left. In the 27-hole circle a 3-degree movement corresponds to 9 holes, and a $\frac{2}{3}$-degree movement to 2 holes, making a total movement of 11 holes in the 27-hole circle, to complete the crank movement for 48 degrees 40 minutes. Below is plainly shown how this calculation may be carried out:

48 deg. 40 min. = 45 deg. +3 deg. +40 min.
5 turns + 9 holes + 2 holes in the 27-hole circle.

Indexing for Minutes. — By using the 18- and 27-hole circles, only whole degrees and $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$ of a degree (20, 30, and 40 minutes) can be indexed. Assume, however, that it is required to index for 16 minutes. One whole turn of the index-crank equals 9 degrees or 540 minutes ($9 \times 60 = 540$). To index for 16 minutes, therefore, requires about $\frac{1}{34}$ of a turn of the index-crank ($540 \div 16 = 34$, nearly). In this case, therefore, an index circle is used having the nearest number of holes to 34, or the index circle with 33 holes. A one-hole movement in this circle would approximate the required movement of 16 minutes.

Assume that it is required to index for 55 minutes. We then have $540 \div 55 = 10$, nearly. In this case there is no index circle with 10 or approximately 10 holes, but as there is an index circle with 20 holes, this circle will be used, and the index-crank is moved two holes in that circle instead of one.

Assume that it is required to index for 2 degrees 46 minutes. If this is changed to minutes, then 2 degrees equals $2 \times 60 = 120$ minutes, and 46 minutes added to this gives a total of 166 minutes. Dividing 540 by 166 gives:

$$540 \div 166 = 3.253.$$

Now multiply this quotient (3.253) by some whole number, so as to obtain a product which equals the number of holes in any one index circle. The number by which to multiply must be found by trial. In this case, 12 can be the multiplier, giving a product of $3.253 \times 12 = 39.036$. For indexing 2 degrees and 46 minutes, the 39-hole circle can, therefore, be used, moving the index-crank 12 holes.

General Rule for Angular Indexing. — The following is a general rule for *approximate* indexing of angles, for any index-head where 40 revolutions of the index-crank are required for one revolution of the work:

Rule: Divide 540 by the total number of minutes to be indexed. If the quotient is approximately equal to the number of holes in any index circle, the angular movement is obtained by moving one hole in this index circle. If the quotient does not approximately equal the number of holes in any index circle, find by trial a number by which the quotient can be multiplied so that the product equals the number of holes in an available index circle; in this circle, move the index-crank as many holes as indicated by the number by which the quotient has been multiplied. (If the quotient of 540 divided by the total number of minutes is greater than the number of holes in any of the index circles, the movement cannot be obtained by simple indexing.)

Rule for Compound Indexing. — The following rule is given for computing the number of holes required for indexing by the compound method:

Rule: Factor the number of divisions required; select two circles of holes at random on the same index-plate for trial, and factor the difference; then draw a line under these two sets of factors. Now factor the number of revolutions required of the index-crank to make one revolution of the index-head spindle, and place these factors under the line. Factor the number of holes in each circle chosen for trial and place these also under the line; then cancel similar factors above and below the line. If all factors above the line cancel, the division is possible with the two circles chosen. The product of the factors remaining below the line will be the number of holes to move forward in one of the circles, and backward in the other.

Example. — Divisions required, 154; circles chosen, 33 and 21; number of turns of crank on index-head, 40. Factoring as mentioned:

$$
\begin{aligned}
154 &= 2 \times 7 \times 11 \\
\underline{33 - 21} &\underline{= 2 \times 2 \times 3 \qquad} \\
40 &= 2 \times 2 \times 2 \times 5 \\
33 &= 3 \times 11 \\
21 &= 3 \times 7
\end{aligned}
$$

After cancelation it will be found that the factors 3 and 5 will remain below the line; hence, $3 \times 5 = 15$ holes, is the number that the index-pin is moved forward in the 21-hole circle and backward in the 33-hole circle to obtain the required division. The mathematical reasoning or the principle upon which this method is founded will now be explained.

Proof of Rule for Compound Indexing. — The method for finding the movements for compound indexing is based upon the principle that the difference between two moves obtainable on the index-plate equals the required division. A certain number of holes moved forward in one circle and the same number of holes moved backward in another circle gives a movement which could not be obtained by either of the index circles selected. To prove the method mathematically, proceed as follows: Let X be the number of holes sought. Other quantities are as in the example given in the preceding paragraph. Now a movement of X number of holes forward in the 21-hole circle and X number of holes backward in the *33-hole circle* equals $\frac{1}{154}$ of a revolution of the work, or:

$$\frac{X}{21 \times 40} - \frac{X}{33 \times 40} = \frac{1}{154};$$

$$\frac{X}{40}\left(\frac{1}{21} - \frac{1}{33}\right) = \frac{1}{154};$$

$$\frac{X}{40}\left(\frac{33 - 21}{21 \times 33}\right) = \frac{1}{154};$$

$$\frac{X}{40 \times 21 \times 33} = \frac{1}{154(33 - 21)}.$$

This shows that if the factors of 40, 21 and 33 are canceled against the factors of 154 and (33 − 21), the factors not canceled in the denominator of the first member, when multiplied, will equal X.

In the given case, after cancelation:

$$\frac{X}{3 \times 5} = 1, \text{ or } \quad X = 15.$$

Another Method of Figuring Compound Indexing Movements. — Suppose it is desired to cut 77 teeth on a gear blank and that an index-plate with a 77-hole circle is not available, so that it is necessary to index by the compound method. We know that the ratio of the worm and worm-wheel is 40 to 1; hence, in order to cut 77 teeth, it is necessary to turn the worm or index-crank through $\frac{40}{77}$ revolution at each setting. The factors of 77 are 7 and 11. The letters x and y will be used temporarily to represent the numbers of holes to be indexed. Then,

$$\frac{x}{7} + \frac{y}{11} = \frac{40}{77}, \tag{1}$$

where $\frac{x}{7}$ = the fraction of a complete turn of the handle for the first indexing movement;

$\frac{y}{11}$ = the fraction of a complete turn of the handle for the second indexing movement.

In this case, the sum of the quantities $\frac{x}{7}$ and $\frac{y}{11}$ equals $\frac{40}{77}$ of a complete turn of the handle. Clearing Formula (1) of fractions:

$$11x + 7y = 40.$$

Then solving the preceding equation for the value of y gives:

$$y = \frac{40 - 11x}{7}.$$

Since it is known that x and y, which represent the numbers of holes for the two indexing movements, must both be integral or whole numbers, various integral values can be assigned to x; these values can be substituted in the preceding expression for the value of x and the expression solved for y, this method of procedure being continued until an integral value of y is found.

When $x = 1$, then y is not a whole number.

When $x = 2$, then y is not a whole number.

When $x = 3$, then $y = 1$.

When the value of 3 is substituted for x, the corresponding value of y is found to be 1, and when these values are inserted in Formula (1), then:

$$\frac{3}{7} + \frac{1}{11} = \frac{40}{77}.$$

As applied to the milling machine index-head, this means that the movement resulting from turning through 3 holes in a 7-hole circle plus the movement resulting from turning through one hole in an 11-hole circle will give the required setting. By multiplying both terms of either fraction by the same number, the value is not changed, and the denominator may be made to assume a value corresponding with the number of holes in a circle which is available on some index-plate. Suppose, for instance, that we multiply both the numerator and the denominator by 3. Then:

$$\frac{9}{21} + \frac{3}{33} = \frac{40}{77}.$$

It will be evident, then, that in order to turn the index-crank $\frac{40}{77}$ of a revolution (thus indexing the work $\frac{1}{77}$ revolution), the index-crank should be moved through 9 holes in a *21-hole circle*, and then through 3 holes in a 33-hole circle.

The plus sign between the fractions indicates that both indexing movements should be in the same direction. In some cases, y will have a minus sign, which indicates that the two indexing movements are in opposite directions. For example, suppose 69 divisions are required. The factors of 69 are 3 and 23; therefore:

$$y = \frac{40 - 23x}{3}.$$

When $x = 1$, then y is not a whole number.

When $x = 2$, then y equals -2. (The algebraic sum of $40 - 23 \times 2 = 40 - 46 = -6$, and $-6 \div 3 = -2$.) Therefore:

$$\frac{2}{3} - \frac{2}{23} = \frac{40}{69}.$$

Multiplying both terms of the fraction $\frac{2}{3}$ by 11, it is changed to $\frac{22}{33}$; hence, the indexing movement is:

$$\frac{22}{33} - \frac{2}{23}.$$

This means that the index-crank is moved forward 22 holes in the 33-hole circle, and backward 2 holes in the 23-hole circle.

CHAPTER XIV

CALCULATIONS FOR CUTTING GEARS

In repair shops or in the smaller manufacturing plants where the cutting of gears is done on a small scale, the milling machine is often used, and it may be desirable or necessary for the one who does the gear-cutting to make whatever calculations are required. This chapter deals with calculations pertaining to spur gears, rack teeth, bevel gears, worm-gears, and spiral gears and includes only the information actually needed in the shop for determining the blank diameters, the depth of the teeth, the angular position of the blank in the case of bevel, spiral and worm-gears, and similar data.

The Depth of Cut for Spur Gears. — The whole depth of a spur-gear tooth, or the depth to which the cutter should be set when milling the teeth, may be determined if either the diametral or circular pitch is known. To obtain the whole depth, divide 2.157 by the diametral pitch, or multiply 0.6866 by the circular pitch.

Example. — If the diametral pitch is 8, the whole depth of the tooth equals $2.157 \div 8 = 0.269$ inch.

Example. — If the circular pitch is 0.3927, the whole depth of the tooth equals $0.6866 \times 0.3927 = 0.2696$ inch.

Pitch of Spur-gear Teeth. — The pitch of the teeth of spur gears may be expressed in two ways. The *circular pitch* is the distance from the center of one tooth to the center of the next along an imaginary circle known as a pitch circle. The *diametral pitch* (which is the term generally employed) represents the number of teeth for each inch of the pitch diameter.

When the circular pitch is known, the diametral pitch is found by dividing 3.1416 by the circular pitch. If the diametral pitch is known, the circular pitch is found by dividing 3.1416 by the diametral pitch.

Example. — If the circular pitch equals 0.3927, then the diametral pitch equals $3.1416 \div 0.3927 = 8$. If the diametral pitch is 12, then the circular pitch equals $3.1416 \div 12 = 0.2618$.

Outside and Pitch Diameters of Spur Gears. — The term "diameter" as applied to a spur gear is generally understood to mean the pitch diameter, or the diameter of the pitch circle, and not the outside diameter. To find the pitch diameter, divide the number of teeth in the gear by the diametral pitch, or multiply the number of teeth by the circular pitch and divide the product by 3.1416.

The outside diameter to which the gear blank is turned may be found by adding 2 to the number of teeth in the gear and dividing the sum by the diametral pitch. The outside diameter may also be determined by adding 2 to the number of teeth and multiplying the sum by the circular pitch and dividing the product by 3.1416.

Example. — If a spur gear is to have 40 teeth of 8 diametral pitch, to what diameter should the blank be turned?

Adding 2 to the number of teeth, and dividing by the diametral pitch, gives $\frac{40 + 2}{8} = 5.25$ inches. Therefore, the outside diameter of this gear, or the diameter to which the blank would be turned, is $5\frac{1}{4}$ inches.

In the case of internal spur gears, the inside diameter to which the gear blank would be bored may be obtained by subtracting 2 from the number of teeth, and dividing the remainder by the diametral pitch.

Diameter of Working Depth and Root Circles. — The following simple rules may be used for obtaining the diameters of the circles representing the working depth of spur-gear teeth and their root diameters.

For the diameter of the circle representing the working depth of the teeth, subtract 2 from the number of teeth in the gear and divide by the diametral pitch; the result is the required diameter.

For the diameter of the root circle, subtract 2.314 from the number of teeth and divide by the diametral pitch; the result is the required diameter.

Example. — Suppose a gear has 48 teeth and is of 4 diametral pitch; then $\frac{48-2}{4} = \frac{46}{4} = 11\frac{1}{2}$ inches, diameter of circle representing the working depth of the teeth. $\frac{48-2.314}{4} = \frac{45.686}{4} = 11.4215$ inches, diameter of circle representing the bottoms of the tooth spaces.

Center-to-center Distance between Spur Gears. — The center-to-center distance between meshing spur gears may be determined by adding the numbers of teeth in both gears and dividing the sum by twice the diametral pitch.

Example. — If one gear has 40 teeth and the other 70 teeth and the diametral pitch is 8, what is the center-to-center distance between the gears?

The total number of teeth in both gears equals $40 + 70 = 110$, and $110 \div 2 \times 8 = 6.875$ inches.

The center distance may also be determined by multiplying the total number of teeth in both gears by the circular pitch and dividing the product by 6.2832. In the case of internal spur gears, the center-to-center distance is found by subtracting the number of teeth in the pinion from the number in the gear and dividing by twice the diametral pitch.

Calculations for Cutting Rack Teeth. — The teeth of a rack are of the same proportions as the teeth of a spur gear or pinion which is intended to mesh with the rack.

Example. — If a pinion having 24 teeth of 6 diametral pitch is to mesh with a rack, what should be the linear pitch of the rack teeth, or the distance from the center of one tooth to the center of the next tooth? How is the whole depth of the rack teeth determined?

The pitch of the rack teeth is equal to the circular pitch of the pinion (distance from the center of one tooth to the center of the next tooth along the pitch circle), and is found by dividing 3.1416 by the diametral pitch. Thus 3.1416 ÷ 6 = 0.5236 inch = linear pitch of rack for meshing with a pinion of 6 diametral pitch. This dimension (0.5236) represents the distance that the cutter would be indexed when milling rack teeth, or the distance that the planer tool would be moved for cutting successive teeth in case the planer were used.

The whole depth of a rack tooth equals 2.157 divided by the diametral pitch of the meshing gear, or the whole depth equals the circular pitch multiplied by 0.6866. In this case, the circular pitch is 0.5236 and the whole depth equals 0.5236 × 0.6866 = 0.3595 inch.

Face Angle of Bevel-gear Blanks with Shafts at Right Angles. — The face angle (see Fig. 1) to which bevel-gear blanks should be turned preparatory to cutting the teeth is found by adding the pitch-cone and addendum angles and subtracting the sum from 90 degrees.

The tangent of the pitch-cone angle of the pinion is found by dividing the number of teeth in the pinion by the number of teeth in the gear. The tangent of the pitch-cone angle of the gear which meshes with the pinion is obtained by dividing the number of teeth in the gear by the number of teeth in the pinion. The tangent of the addendum angle for both the gear and the pinion is found by dividing the addendum by the pitch-cone radius.

To find the addendum of both the gear and the pinion, divide 1 by the diametral pitch, or multiply the circular pitch by 0.318.

To find the pitch-cone radius (C, Fig. 1) divide the pitch diameter by twice the sine of the pitch-cone angle, and to find the pitch diameter, divide the number of teeth by the diametral pitch, or multiply the number of teeth by the circular pitch and divide by 3.1416.

Example. — A pair of bevel gears of 3 diametral pitch are to be mounted upon shafts at right angles to each other. The gear is to have 60 teeth, and the pinion 15. What is the face angle of the gear, or the angle at which the compound rest of the lathe would be set for turning the blank?

The tangent of the pitch-cone angle equals 60 ÷ 15 = 4, which is a tangent of 75 degrees 58 minutes. In order to determine the addendum angle, it is necessary to find the addendum, the pitch diameter, and the pitch-cone radius.

The addendum = 1 ÷ 3 = 0.3333;
The pitch diameter = 60 ÷ 3 = 20 inches;
The pitch-cone radius

$$= 20 \div (2 \times \sin 75^\circ 58') = \frac{20}{2 \times 0.97015} = 10.3077 \text{ inches.}$$

The tangent of the addendum angle equals the addendum divided by the pitch-cone radius, or 0.3333 ÷ 10.3077 = 0.0323, which is the tangent of 1 degree 51 minutes.

Having now determined the pitch-cone and addendum angles, the face angle equals 90 degrees − (75° 58′ + 1° 51′) = 12 degrees 11 minutes. Therefore, when turning the blank from which the gear is to be made, the compound rest should be swiveled around 12⅛ degrees from its zero position.

In order to calculate the face angle of the pinion, its pitch-cone angle must first be determined. The tangent of the pitch-cone angle for the pinion equals the number of teeth in the pinion divided by the number of teeth in the gear; in this case, 15 ÷ 60 equals 0.25, which is the tangent of 14 degrees 2 minutes. The addendum angle for the pinion is the same as for the gear, and, therefore, the face angle equals 90 degrees − (14° 2′ + 1° 51′) = 74 degrees 7 minutes. The compound rest, then, should be set to 74⅛ degrees, approximately, for turning the pinion blank.

Outside Diameter of a Bevel-gear Blank. — The outside diameter (*O*, Fig. 1) of a bevel-gear blank is obtained as follows: First, multiply the addendum by the cosine of the pitch-cone angle; then multiply this product by 2 and add the product thus obtained to the pitch diameter. Take as an example the bevel gearing referred to previously. The gear has 60 teeth and the pinion 15 teeth of 3 diametral pitch, and the shafts are at right angles to each other. What is the outside diameter of the gear?

The tangent of the pitch-cone angle of the gear equals the number of teeth in the gear divided by the number of teeth in the pinion, or 60 ÷ 15 = 4 = tangent 75 degrees 58 minutes. The addendum equals 1 divided by the diametral pitch, or 1 ÷ 3 = 0.3333. The addendum, or 0.3333 × cosine 75 degrees 58 minutes = 0.3333 × 0.24249 = 0.0808, and 0.0808 × 2 + pitch diameter = outside diameter. The pitch

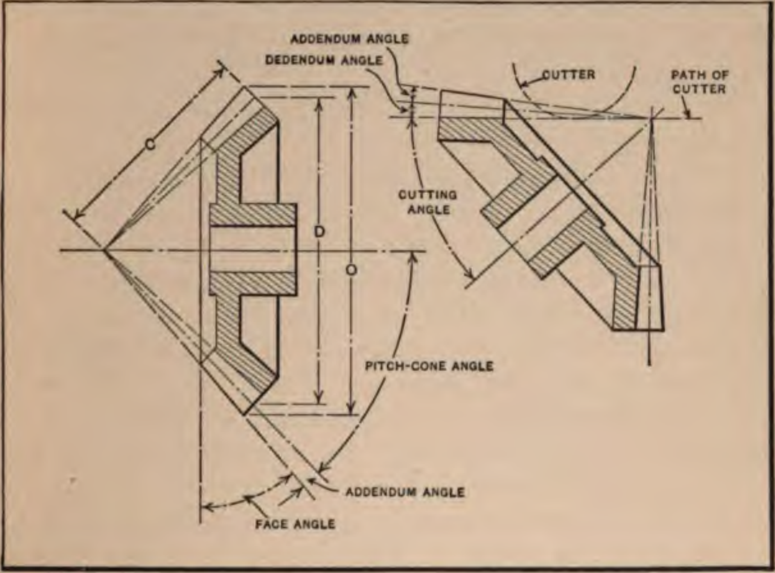


Fig. 1. Angles and Other Values required in the Solution of Bevel-gear Problems

diameter equals the number of teeth divided by the diametral pitch, or 60 ÷ 3 = 20. Therefore, the outside diameter equals 0.0808 × 2 + 20 = 20.161 inches.

Angular Position of Bevel Gear for Cutting Teeth. — The angle at which a bevel gear is set for cutting the teeth is usually obtained by subtracting the dedendum angle (see Fig. 1) from the pitch-cone angle, although when the teeth are to be milled with a formed cutter it is considered preferable to subtract the addendum angle from the pitch-cone angle instead of the dedendum angle, as a more uniform clearance is obtained at the bottom of the tooth spaces and a somewhat closer approximation to the correct tooth shape. The tangent of the dedendum angle equals the dedendum divided by the pitch-cone radius. The tangent of the dedendum angle equals the dedendum divided by the pitch-cone radius.

Example. — The gear referred to in a preceding example is to have 60 teeth of 3 diametral pitch, and is to mesh with a pinion having 15 teeth, the shafts being at right angles to each other. What is the cutting angle for the gear, assuming that the dedendum angle is to be subtracted from the pitch-cone angle?

The dedendum equals 1.157 divided by the diametral pitch, or $1.157 \div 3 = 0.3856$ inches.

The pitch diameter equals the number of teeth divided by the diametral pitch, or $60 \div 3 = 20$ inches.

The tangent of the pitch-cone angle of the gear equals the number of teeth in the gear divided by the number of teeth in the pinion, or $60 \div 15 = 4$, which is the tangent of 75 degrees 58 minutes. (In the case of miter gears, the pitch-cone angle equals 45 degrees.)

The pitch-cone radius (C, Fig. 1) equals the pitch diameter divided by twice the sine of the pitch-cone angle, equals $20 \div (2 \times 0.97015) = 10.3077$ inches.

The dedendum angle equals the dedendum divided by the pitch-cone radius equals $0.3856 \div 10.3077 = 0.0374$, which is a tangent of 2 degrees 9 minutes. Therefore, the cutting angle equals 75 degrees 58 minutes − 2 degrees 9 minutes = 73 degrees 49 minutes, or, approximately, $73\frac{5}{6}$ degrees.

Number of Teeth for which to Select Bevel-gear Cutter. — When the teeth of bevel gears are milled with a formed cutter, the number of cutter to use for a given pitch depends upon the number of teeth in the bevel gear and its pitch-cone angle. If the actual number of teeth in the gear is divided by the cosine of the pitch-cone angle, the result will equal the number of teeth for which to select a cutter.

Example. — If a bevel gear has 60 teeth of 3 diametral pitch, and a pitch-cone angle of 75 degrees 58 minutes, for what number of teeth should the cutter be selected?

The cosine of 75 degrees 58 minutes equals 0.24249, and 60 ÷ 0.24249 = 247. Therefore, a cutter of 3 diametral pitch and No. 1 shape would be used, because the No. 1 cutter is intended for all numbers of teeth from 135 to a rack.

Calculations for Cutting Worm-gearing. — As a practical example to illustrate calculations relating to worm-gearing, assume that a worm having an outside diameter of $2\frac{1}{2}$ inches and a double thread of $\frac{1}{2}$ inch linear pitch is to be cut so as to mesh with a worm-wheel having 45 teeth.

The lead of the thread for which the lathe must be geared when cutting the worm equals the pitch times 2 for a double-threaded worm, the pitch times 3 for a triple-threaded worm, and so on. In this case, the lead equals $0.5 \times 2 = 1$ inch. The whole depth W of the worm-thread (see Fig. 2) equals the linear pitch $\times$ 0.6866, or $0.5 \times 0.6866 = 0.3433$ inch.

The bottom diameter B equals the outside diameter minus twice the whole depth of the thread, or $2.5 - 2 \times 0.3433 = 1.8134$ inch. The angle of the threading tool for worm-gearing is 29 degrees. These calculations cover requirements for the worm itself, as far as cutting it is concerned.

Throat Diameter of Worm-wheel Blank. — The blank from which the worm-wheel is to be made must be turned to the correct diameter, and most worm-wheels have a curved throat, the radius of which must be determined. Continuing with the example given at the beginning of the preceding paragraph, it is necessary to determine the throat diameter T (Fig. 2) of the worm-wheel, and the radius R of the throat.

The throat diameter is found by adding twice the addendum of the worm-thread to the pitch diameter of the worm-wheel. The addendum of the worm-thread equals the linear pitch multiplied by 0.3183, and in this case equals $0.5 \times 0.3183 = 0.1591$ inch.

The pitch diameter of the worm-wheel is obtained by multiplying the number of teeth in the wheel by the linear pitch of the worm, and dividing the product by 3.1416 equals $45 \times 0.5 \div 3.1416 = 7.162$ inches. Hence, the throat diameter equals $7.162 + 2 \times 0.1591 = 7.48$ inches.

Minimum Length of Worm. — The shortest length to which the worm should be made is found by subtracting four times the addendum of the worm-thread from the throat diameter of the worm-wheel, squaring the remainder, and subtracting the result from the square of the worm-wheel throat diameter. The square root of the result represents the minimum length of the worm.

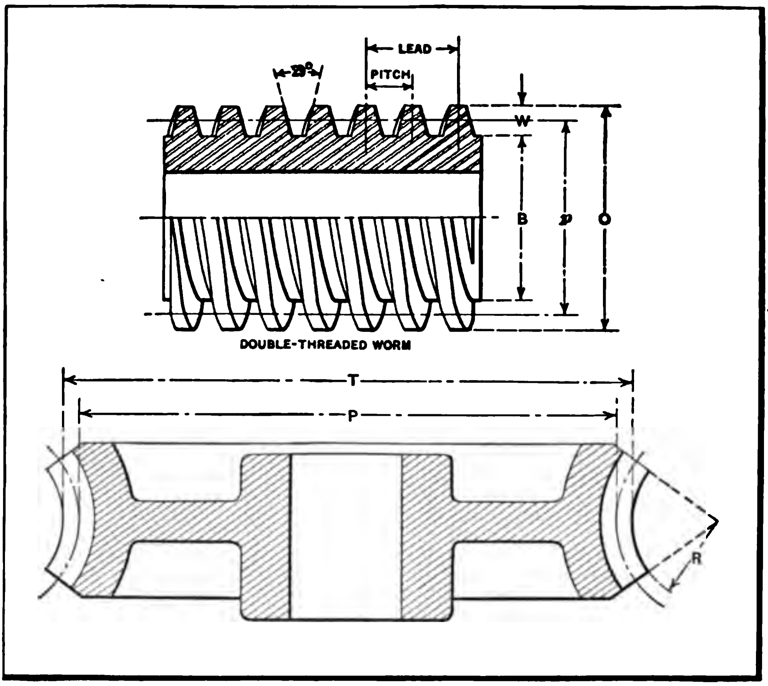


Fig. 2. Double-threaded Worm — Sectional View of Worm-wheel

Taking the example referred to in the preceding paragraph, the addendum of the worm having a linear pitch of 0.5 inch equals 0.1591 inch, and the throat diameter of the worm-wheel equals 7.48 inches. Four times the addendum subtracted from the throat diameter equals $7.48 - (4 \times 0.1591) = 6.843$ inches. This remainder squared and subtracted from the square of the throat diameter equals $7.48^2 - 6.843^2 = 9.12$ square inches. The square root of 9.12 equals 3 inches nearly, which represents the shortest length of worm to obtain complete action with the worm-wheel.

Radius of Worm-wheel Throat. — The radius R (see Fig. 2) of a worm-wheel throat is found by subtracting twice the addendum of the worm-thread from one-half the outside diameter of the worm. The addendum of the worm-thread (as previously explained) equals $0.5 \times 0.3183 = 0.1591$ inch, and radius $R = (2.5 \div 2) - 2 \times 0.1591 = 0.931$ inch.

Angular Position of Worm-wheel for Gashing. — When a worm-wheel is hobbed in a milling machine, gashes are milled before the hobbing operation. The table of the machine, while gashing, must be swiveled around from its right-angle position, the amount depending upon the relation between the lead of the worm-thread and the pitch circumference.

Example. — If a worm-wheel is to mesh with a double-threaded worm having a linear pitch of 0.5 inch and an outside diameter of 2.5 inches, at what angle should the milling machine table be set for gashing?

The first step is to find the circumference of the pitch circle of the worm. The pitch diameter equals the outside diameter minus twice the addendum of the worm thread, and the addendum equals the linear pitch times 0.3183, or $0.5 \times 0.3183 = 0.1591$ inch. Hence, the pitch diameter equals $2.5 - 2 \times 0.1591 = 2.18$ inches, and the pitch circumference equals $2.18 \times 3.1416 = 6.848$ inches.

As the worm is double-threaded, the lead equals 2 times the pitch, or 1 inch. After determining the lead and the pitch circumference, the angle to which the table of the machine should be set is found as follows:

Rule: Divide the lead of the worm-thread by the pitch circumference to obtain the tangent of the desired angle, and then refer to a table of tangents to determine what this angle is. In this case it is $1 \div 6.848 = 0.1460$, which is the tangent of $8\frac{1}{3}$ degrees, nearly. Therefore, the table of the milling machine is set at an angle of $8\frac{1}{3}$ degrees from its normal position.

Pitch of Cutter for Spiral Gears. — If the number of teeth in a helical or spiral gear, the helix angle of the teeth relative to the axis, and the pitch diameter are known, the diametral pitch of the cutter to use for that gear may be determined as follows:

Rule: Divide the number of teeth by the pitch diameter, thus obtaining the "real diametral pitch." The "normal diametral pitch," which represents the pitch of the cutter to use, is then found by dividing the real diametral pitch by the cosine of the helix angle of the gear teeth.

Example. — Assume that the gear has 48 teeth, a pitch diameter of 12.5 inches, and a tooth angle of 16 degrees 16 minutes. What is the normal diametral pitch of the cutter? The real diametral pitch equals $48 \div 12.5 = 3.84$. The cosine of 16 degrees 16 minutes is 0.9599; therefore, the normal diametral pitch of the cutter equals $3.84 \div 0.9599 = 4$, very nearly; hence, a cutter of 4 diametral pitch would be used. This same cutter could be used for a spur gear having 48 teeth and a pitch diameter of 12 inches.

Depth of Cut for Spiral Gears. — After the normal diametral pitch of the cutter to use for spiral gears is determined, as explained in the preceding paragraph, the whole depth of the tooth, or depth to which the cutter should be sunk into the blank, may be found by dividing 2.157 by the normal diametral pitch. If the normal diametral pitch were 4, the whole depth of tooth would equal $2.157 \div 4 = 0.539$ inch.

Cutter Number for Spiral Gears. — The formed cutters sometimes used for cutting spur gears may also be employed for spiral gears, although the number of the cutter for a given pitch is not selected with reference to the actual number of teeth in a spiral gear as in the case of a spur gear.

Rule: When a spiral gear is to be milled with a formed cutter, the number of teeth for which the cutter should be selected may be determined by dividing the actual number of teeth in the spiral gear by the cube of the cosine of the tooth angle.

Example. — If the angle between the teeth and the axis of a spiral gear is 20 degrees, and the gear is to have 48 teeth, what number of cutter should be used, assuming that involute formed gear-cutters made according to the Brown & Sharpe system for spur gears are to be used?

The cosine of 20 degrees is 0.9397, nearly. Therefore, the number of teeth for which to select a cutter equals $\frac{48}{(0.9397)^3}$ = 57.8 or, say, 58 teeth. The No. 2 cutter would be used in this case, as this is intended for gears having from 55 to 134 teeth.

Lead of Spiral-gear Teeth. — If a spiral or helical gear were wide enough, the teeth would wind around the gear like the threads of a multiple-threaded screw. When cutting a spiral gear, it is necessary to determine the lead of the teeth, or the distance each tooth would advance, assuming that it made one complete turn. If a milling machine is used for cutting the gear, the index-head and table feed-screw are connected by change-gears selected according to the lead. When the pitch diameter and tooth angle (angle between a tooth and the axis of the gear) are known, the lead of the teeth may be found as follows:

Rule: Multiply the pitch circumference of the gear by the cotangent of the tooth angle. The lead is equal to the pitch circumference when the tooth angle is 45 degrees.

Example. — If the pitch diameter equals 4.005 inches, and the tooth angle is $22\frac{1}{2}$ degrees, what is the lead of the gear teeth?

The first step is to find the pitch circumference, which equals $4.005 \times 3.1416 = 12.582$ inches. As the cotangent of $22\frac{1}{2}$ degrees is 2.414, the lead equals $12.582 \times 2.414 = 30.373$ inches.

In designing spiral gears, the diameter and helix angle of the teeth are ordinarily made to suit conditions so that the lead of the teeth may be an odd dimension which cannot be obtained exactly, although usually some combination of the change-gears furnished with a universal milling machine will give a lead which is accurate enough for practical purposes.

Calculating Tooth Angle of Spiral Gear from Sample. — It is sometimes necessary to determine the helix angle between the teeth of a spiral gear and its axis, by measuring a gear which is already cut, the object being to reproduce the sample gear. In order to determine the helix angle by the following method, it is necessary to know the width W of the face (see Fig. 3), the distance A that a tooth advances in the width of

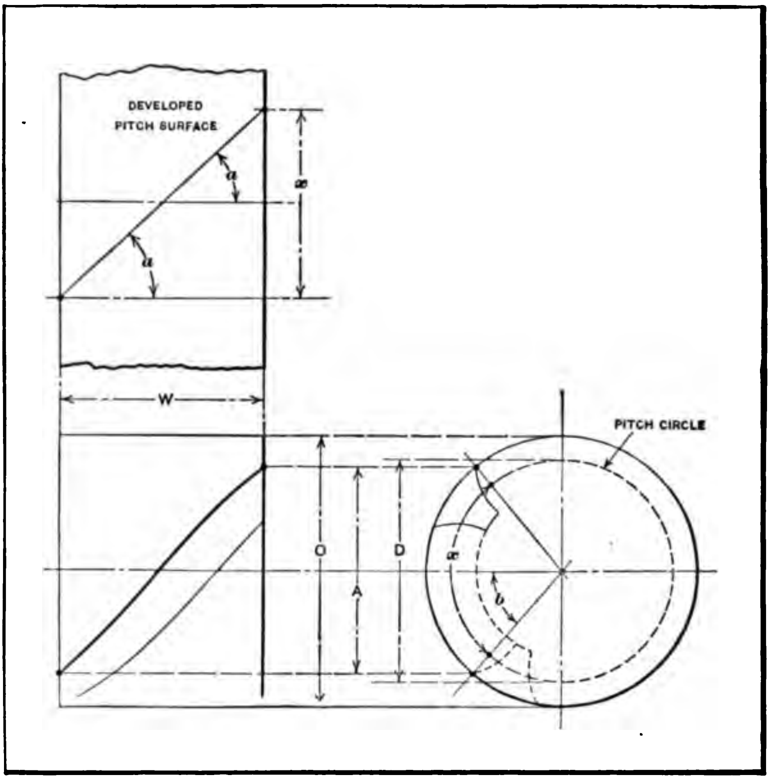


Fig. 3. Diagram illustrating Method of calculating Tooth Angle of Spiral Gear from Sample

the face, the outside diameter O, and the pitch diameter D. All of these measurements are taken directly from the sample, except the pitch diameter. The latter is obtained from the normal diametral pitch of the cutter used for cutting the gear teeth, and is found as follows: Divide 2 by the normal diametral pitch, and subtract the quotient from the outside *diameter.* The difference equals the pitch diameter.

Suppose the outside diameter of the gear is 6.968 inches and the normal diametral pitch of the cutter used for cutting the sample gear is 8 inches. Then the pitch diameter of the gear equals 6.968 − (2 ÷ 8) = 6.718 inches, which is the pitch diameter.

The next step is to calculate the distance x (Fig. 3) which the tooth advances on the pitch circle in the width of the face. The advance A is first measured by using a height gage, or in any other convenient way. The sine of angle b equals dimension A divided by the outside diameter. Thus:

$$\text{Sin } b = \frac{A}{2} \div \frac{o}{2} = \frac{A}{o}.$$

Assuming that A measures 3.01 inches, then sine b equals $\frac{3.01}{6.968} = 0.432$. By referring to a table of sines, it is found that angle b equals 25 degrees 36 minutes, or 25.6 degrees.

When this angle has been determined, the dimension x can be found by proportion, thus:

$$\frac{x}{2} : 3.1416 \times D :: 25.6 \text{ degrees} : 360 \text{ degrees}.$$

This proportion may be simplified to the following expression:

$$x = \frac{25.6 \times 3.1416 \times D}{180} = 3.0016 \text{ inches}.$$

After determining the distance x which the helix advances along the pitch circle in the width of the gear, the helix angle may be found easily. This helix angle a is indicated in Fig. 3 on the development at the upper part of the illustration, which represents a section of the surface of the pitch cylinder as it would appear if the surface had been laid out flat on a drawing-board. The tangent of helix angle a equals dimension x divided by the width W of the gear. Assuming this width to be 3 inches, then the tangent of angle a equals 3.0016 ÷ 3 = 1.0005, which is the tangent of 45 degrees 1 minute, thus indicating that the helical angle of the gear is 45 degrees.

CHAPTER XV

TYPICAL MACHINE SHOP PROBLEMS

THE use of mathematics in the machine shop and tool-room often saves time and prevents spoiling work by substituting a direct and accurate method for the "cut-and-try" method which is frequently resorted to by those who are unable to apply mathematical principles to the problems at hand. While the drafting-room is the proper place to do the mathematical work, a great many machinists and toolmakers, particularly in small shops, find it necessary to carry on certain operations independently of a draftsman and sometimes without a drawing when, as often occurs in repair work, drawings do not exist. Some of the examples included in this chapter are intended to illustrate principles which have been explained in preceding chapters. Other examples are inserted primarily because they represent the kind of information frequently required in general machine shop practice.

Pitch and Lead of Screw Threads and Number of Threads per Inch.— The terms pitch and lead of screw threads are often confused. The pitch of a screw thread is the distance from the top of one thread to the top of the next thread. No matter whether the screw has a single, double, triple, or quadruple thread, the pitch is always the distance from the top of one thread to the top of the next thread. The lead of a screw thread is the distance the nut will move forward on the screw, if it is turned around one full revolution. In the single-threaded screw, the pitch and lead are equal, because the nut would move forward the distance from one thread to the next, if turned around once. In a double-threaded screw, however, the nut will move forward two threads, or twice the pitch, so that, in a double-threaded screw, the lead equals *twice the* pitch. In a triple-threaded screw, the lead equals three times the pitch, and so forth. The lead may also be expressed as being the distance from center to center of the *same* thread, after one turn.

The word pitch is often, though improperly, used in the shop to denote the number of threads per inch. Screws are often referred to as having 12 pitch thread, 16 pitch thread, when 12 threads per inch and 16 threads per inch is what is really meant. The number of threads per inch equals 1 divided by the pitch, or, expressed as a formula:

$$\text{Number of threads per inch} = \frac{1}{\text{pitch}}.$$

The pitch of a screw equals 1 divided by the number of threads per inch, or:

$$\text{Pitch} = \frac{1}{\text{number of threads per inch}}.$$

Thus, if the number of threads per inch equals 16, the pitch equals $\frac{1}{16}$. If the pitch equals 0.05, the number of threads per inch equals $1 \div 0.05 = 20$. If the pitch equals $\frac{3}{8}$ inch, the number of threads per inch equals $1 \div \frac{3}{8} = 2\frac{2}{3}$.

Confusion is often caused by indefinite designation of multiple-thread (double, triple, quadruple, etc.) screws. One way of expressing that a double-thread screw is required is to say, for instance: "3 threads per inch double," which means that the screw is cut with 3 double threads, or 6 threads per inch. The pitch of this screw is $\frac{1}{6}$ inch, and the lead is twice this, or $\frac{1}{3}$ inch. To cut this screw, the lathe will be geared to cut 3 threads per inch, but the thread will be cut only to the depth required for 6 threads per inch. "Four threads per inch triple" means that there are 4 times 3, or 12 threads along one inch of the screw. The pitch of the screw is $\frac{1}{12}$ inch, but being a triple screw, the lead of the thread is 3 times the pitch, or $\frac{1}{4}$ inch.

The best way of expressing that a multiple-thread screw is to be cut, when the lead and the pitch have been figured, is, for example: "$\frac{1}{4}$ inch lead, $\frac{1}{12}$ inch pitch, triple thread." In the case of single-threaded screws, the number of threads per inch and the form of the thread only are given. The word "single" is not required.

Diameters of Holes before Tapping and Thread Cutting. — Holes that are to be tapped should ordinarily be somewhat larger than the root diameter of the thread to reduce tap breakage and to increase the tapping speed. A thread equal to about 75 per cent of the standard depth is considered about right for ordinary manufacturing practice. If the holes are to have U. S. standard threads, the diameter of the tap drill may be determined as follows:

Rule: Divide 1.299 by the number of threads per inch, multiply the quotient by 0.75, and subtract the result from the outside diameter of the tap. To allow for a full depth of thread, the multiplication by 0.75 before subtracting from the outside diameter is omitted. If some other percentage of depth is desired, this is merely substituted for the figure given.

Example. — What is the diameter of a tap drill for holes to be tapped with $\frac{13}{16}$ U. S. standard threads?

The $\frac{13}{16}$ U. S. standard thread has 10 threads per inch.

$\frac{1.299}{10} = 0.1299$, and $0.1299 \times 0.75 = 0.0974$ inch.

The outside diameter of the tap, or $0.8125 - 0.0974$, equals 0.715 inch. Hence, a drill $\frac{45}{64}$ inch in diameter should be used.

In figuring the tap drill sizes for other threads, the following constants are used:

For a sharp V-thread, divide 1.732 by the number of threads per inch.

For the Whitworth standard thread, divide 1.2806 by the number of threads per inch.

When cutting internal threads in a lathe, the hole is usually bored to the root diameter, especially for the larger sizes of screw threads, instead of making the diameter somewhat larger than the root diameter, as when tapping. To obtain the root diameter, divide one of the constants previously given by the number of threads per inch and subtract the *quotient from* the outside diameter of the screw thread.

Example. — If a hole is to be threaded to receive a U. S. standard screw 4 inches in diameter, to what diameter should the hole be bored?

The 4-inch U. S. standard screw thread has 3 threads per inch; hence the root diameter equals $4 - \frac{1.299}{3} = 4 - 0.433 = 3.567$ inches.

Inclination of Thread Tool Relative to Thread. — When cutting screw threads, and especially square threads which incline considerably relative to the axis, it is desirable, if not necessary, to set the cutting end of the tool in line with the thread groove, or at the helix angle. The diagram (Fig. 1) illustrates how tapering steel strips may be used for holding the tool in the proper position. The angle *a* of these strips is the same as the helix angle of the thread relative to a plane perpendicular to the axis of the screw. The tangent of this angle equals the lead of the thread divided by the circumference of the screw. The outside circumference may be taken in most cases, although the pitch circumference is more nearly correct.

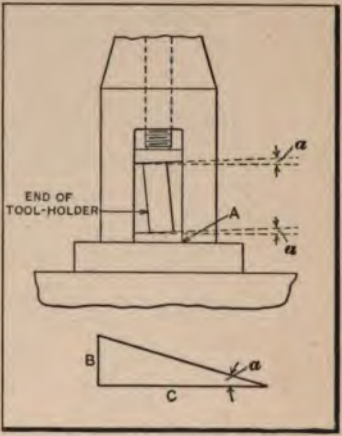


Fig. 1. Thread-cutting Tool clamped between Taper Strips for Holding it in Alignment with the Thread Groove

Example. — A number of double-threaded screws $2\frac{1}{4}$ inches in diameter and $\frac{1}{4}$ inch pitch are to be cut, and it is desired to make tapering strips for holding the thread tool in line with the thread groove. To what angle *a* (Fig. 1) should the strips be made?

Since the screw is doubled-threaded, it is necessary to consider the lead of the thread in order to determine the proper helix angle. The lead of a double thread equals twice the pitch, or, in this case, $0.25 \times 2 = 0.5$ inch. The outside circumference of the screw equals $2.25 \times 3.1416 = 7.068$. Therefore, $\tan a = \frac{0.5}{7.068} = 0.0707$, which is the tangent of 4 degrees, approximately.

Width of Cutter for Milling Straight-tooth Clutches. — The width of the cutter to use for milling the teeth of straight-tooth clutches (of the type illustrated by the diagram, Fig. 2) depends upon the width of the tooth space across the inside diameter of the clutch.

Rule: In order to determine the cutter width W, first find the angle a of the tooth space by dividing 360 by twice the number of clutch teeth. Divide the sine of this angle by 2 and multiply the quotient by the inside diameter D of the clutch to obtain the width W of the cutter.

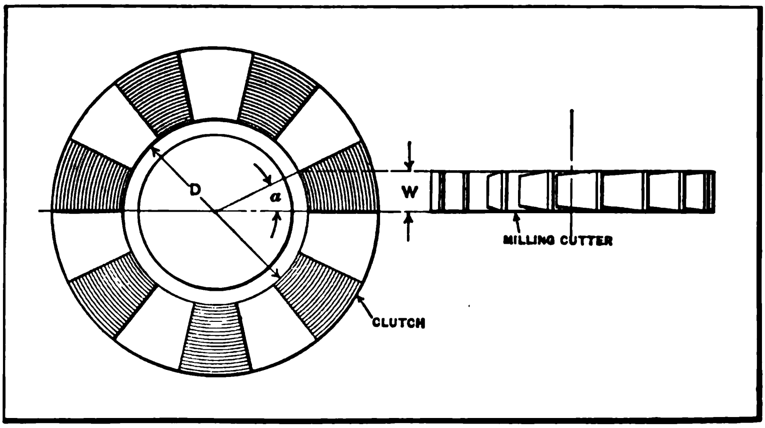


Fig. 2. Diagram illustrating Problem of determining Width *W* of Cutter for Milling Straight-tooth Clutch

Example. — If the clutch is to have 7 teeth and the inside diameter D is 4 inches, what width of cutter should be used?

As there are to be 7 teeth, the angle a of the tooth space equals $360 \div 14 = 25.7$, or 25 degrees 40 minutes, approximately. The sine of 25 degrees 40 minutes, or $0.433 \div 2 = 0.216$, and $0.216 \times 4 = 0.86$, or $\frac{7}{8}$ inch, nearly. Hence, a cutter $\frac{7}{8}$ inch wide would be used.

Clutches of this kind usually have an odd number of teeth, the advantage being that a cut can be taken clear across the blank when milling teeth, thus finishing the sides of two teeth at one passage of the cutter.

Angular Position of Blank for Milling Saw-tooth Clutches. — When the teeth of saw-tooth clutches are being milled, the clutch blank should be set at such an angle that the bottoms and tops of the teeth incline an equal amount relative to the axis of the clutch. The teeth on the driving and the driven parts of the clutch will then mesh properly. The diagram (Fig. 3) represents the clutch blank held in an angular

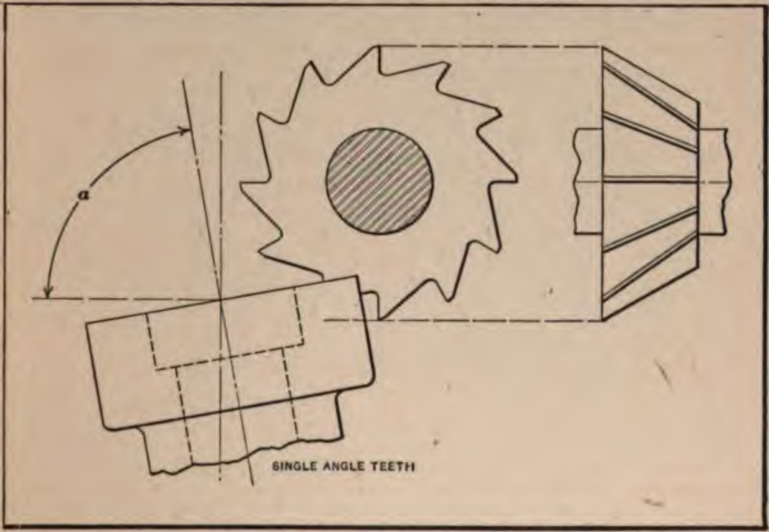


Fig. 3. Diagram illustrating Problem of determining Angle *a* to which Saw-tooth Clutch is set when Milling Teeth

position by the dividing-head of a milling machine. The angle *a* to which the dividing-head should be set may be determined as follows:

Rule: Divide 180 by the number of teeth the clutch is to have; find the tangent of the angle thus obtained and multiply this tangent by the cotangent of the cutter angle. The result equals the cosine of angle *a* (Fig. 3) to which the dividing-head should be set. This rule is clearer when expressed as a formula. Thus:

$$\text{Cos } a = \tan \frac{180^\circ}{N} \times \text{cot cutter angle.}$$

In this formula, N equals the number of teeth in the clutch. Both the formula and the preceding rule apply to the single-angle form of clutch-tooth which has one side parallel with the axis of the clutch.

Example. — A clutch is to have 10 teeth and is to be milled with a 60-degree single-angle cutter. To what angle a should the dividing-head be set?

$180 \div 10 = 18°$, and the tangent of 18 degrees is 0.3249. The cotangent of the cutter angle equals 0.5773, and $0.5773 \times 0.3249 = 0.1875$, which is the cosine of 79 degrees 10 minutes, nearly.

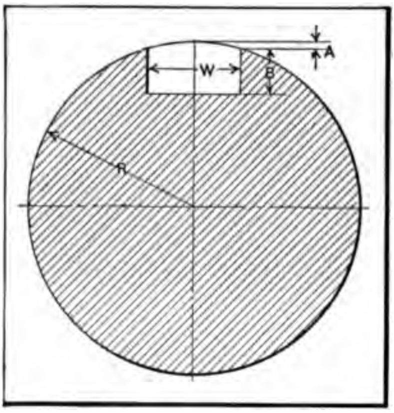


Fig. 4. Values required in Formula for Keyway Milling

Depth of Keyway.—When milling or planing a keyway, it is convenient to adjust the tool until it just grazes the top of the shaft, and then adjust it to the proper depth from this point. If the depth B (Fig. 4) at one side is given on the drawing, it is necessary to determine the height of the arc A and add this to depth B to determine the total depth as measured from the top of the shaft.

Rule: To find the height of arc A, square one-half the width of the keyway and subtract the result from the square of the shaft radius. The square root of the remainder, subtracted from the shaft radius, equals the height A of the arc.

Expressed as a formula in which the letters correspond to those in Fig. 4:

$$A = R - \sqrt{R^2 - (\tfrac{1}{2}W)^2}.$$

Example. — If the shaft diameter is 10 inches and the width of the keyway is 2 inches, what is the height of arc A?

$$A = 5 - \sqrt{5^2 - 1^2} = 5 - \sqrt{24} = 5 - 4.899 = 0.101 \text{ inch.}$$

This amount should be added to the depth B of the keyway to obtain the total depth. When milling the keyway, the cutter would be set to just graze the top of the shaft, and then the knee of the machine should be elevated an amount equal to this total depth.

Taper of Dovetail Slide for Given Gib Taper. — In planing taper gibs and dovetail slides, if the gib is first planed to a given taper per foot the dovetail slide should not be planed to this same taper per foot, assuming that the taper on the

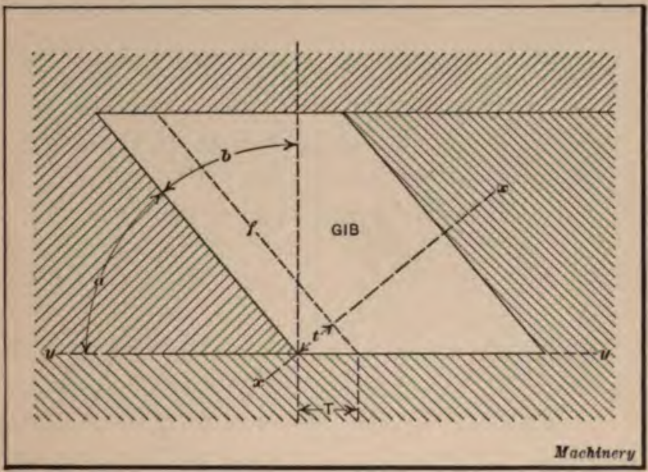


Fig. 5. Illustrating Method of determining Taper of Dovetail Slide for Given Gib Taper

gib is measured in a plane $x - x$ (see Fig. 5) and the taper of the slide is measured in a plane $y - y$. Suppose that the gib as first planed is simply a tapering strip of rectangular cross-section, and that it has a taper of 0.125 inch per foot. After the top and bottom edges of the gib are planed so that they are parallel with plane $y - y$ when the gib is in place, the next problem is to determine the taper of the slide. The dotted line f in the illustration represents the taper of the gib in a length of one foot. This taper will equal dimension t if measured in plane $x - x$, and in this case is 0.125 inch. Now, in order to set the slide in the proper position on the machine, the amount of taper T measured in plane $y - y$

is first determined and then the corresponding angle. This taper T depends upon the angle of the dovetail and represents the taper to which the slide should be planed in order to have the outer side of the gib parallel with the opposite or straight side of the dovetailed part.

If the angle a is 50 degrees, the angle b equals $90 - 50 = 40$ degrees. Having the angle b, the taper T for a given taper t equals $\frac{t}{\cos b}$. Expressing this formula as a rule, the taper for planing a dovetail to fit a tapering gib may be obtained as follows:

Rule: Subtract the angle a of the dovetail slide from 90 degrees and find the cosine corresponding to the difference between these angles. Divide the taper of the gib by this cosine to obtain the taper of the dovetail in a plane $y - y$.

In this case, the angle b equals 40 degrees and the cosine 40 degrees equals 0.766. The taper to which the gib is planed equals 0.125 inch per foot, and $0.125 \div 0.766 = 0.163$, which is the taper per foot in a plane $y - y$.

The tangent of the angle corresponding to the taper per foot may be found by dividing the taper in inches per foot by 12. In this case, the taper per foot of the dovetail slide is 0.163 inch, when measured parallel to the flat bearing surfaces. At what angle should the slide be set for planing or milling that side against which the gib bears?

The tangent of the angle equals $0.163 \div 12 = 0.0136$ which is the tangent of 47 minutes. Therefore, one side of the slide should be set as nearly as possible to this angle when machining the opposite or gib side. It will be understood that the angular position of the slide when planing or milling it, is relative to the travel of the machine table.

Finding Lead of Spiral Milling Cutter from Sample. — The lead of a spiral milling cutter, like the one shown by the diagram, Fig. 6, may be determined from the cutter itself by the following method:

First, it is necessary to obtain the angle between the teeth and the axis of the cutter. A practical method of finding this angle is to coat the land or edge of one of the teeth, as AB, with some marking material, such as black lead, and then roll the cutter upon a flat sheet of paper in the direction shown by arrow K. The contact between the spiral or, more properly, helical edge of the cutter tooth and the paper will form a straight line A_1B_1. In order to determine the angle, it may be preferable to extend this line by means of a straightedge. Another line M_1N_1 is then drawn, to represent the axis of the cutter at right angles to line C along which the cutter was rolled. The next step is to measure angle a by means

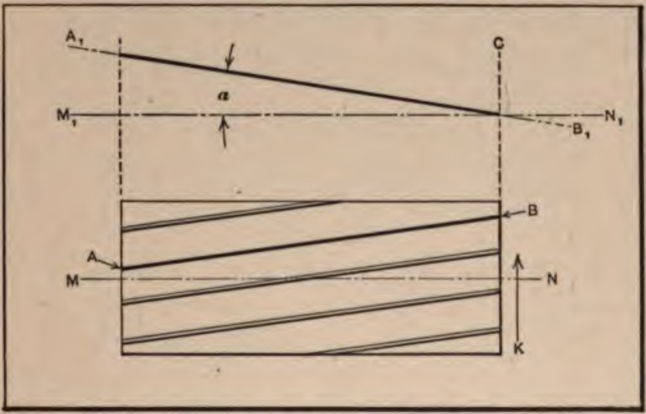


Fig. 6. Method of determining Angle between Teeth and Axis of Spiral Milling Cutter

of a protractor. The lead of the spiral or helix for any circumference is then determined by multiplying this circumference by the cotangent of the helix angle just measured. For instance, if the circumference were 12 inches and the cotangent of angle a were 4, the lead of the spiral would be 12 times 4, or 48 inches. The cutter might be rolled along one edge of the sheet, or, all of the teeth could be covered at one end with marking material to locate line C.

Calculations for Measuring Dovetail Slides. — A common method of measuring or gaging dovetail slides is illustrated by Fig. 7. The upper view, A, shows how a male dovetail is gaged by measuring the distance x across two cylindrical rods or plugs which are placed against each side of the dovetail. This same general method applied to a female dovetail is illustrated by the lower view, *B*. The dimension *x* for gaging the male dovetail may be determined by the following rule:

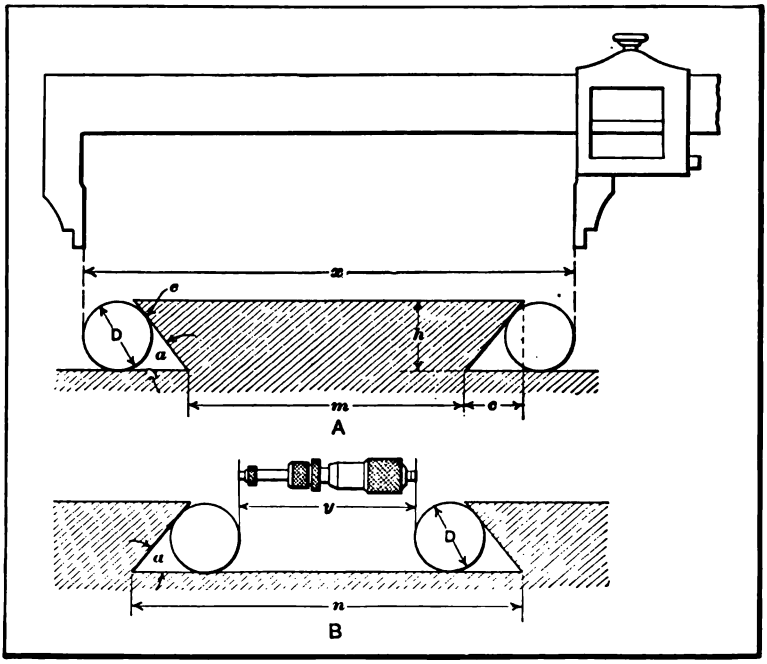


Fig. 7. Measurement of Dovetail Slides by Cylindrical Plug Method

Rule: Add one to the cotangent of one-half the dovetail angle *a* and multiply the sum by the diameter *D* of the cylindrical rods used. Next add the product thus obtained to dimension *m*.

Expressing this rule as a formula:

$$x = D \times (1 + \cot \tfrac{1}{2} a) + m.$$

Example. — If the width *m* of a male dovetail is to be 10 inches, the angle *a*, 50 degrees, and diameter *D* of the rods, 1.25 inch, to what dimension *x* should a vernier caliper be set for testing the width of the dovetail?

Dimension $x = 1.25 \times (1 + \cot\ 25\ \text{degrees}) + m = 1.25 \times 3.1445 + 10 = 13.93$ inches.

When the female dovetail is to be measured, as illustrated at *B*, dimension *y* may be found by the following rule:

Rule: Add one to the cotangent of one-half the dovetail angle *a*; multiply the sum by the diameter *D* of the rod, and subtract the result from dimension *n*.

Expressing this rule as a formula,

$$y = n - D \times (1 + \cot \tfrac{1}{2} a).$$

If the dimension *c* is required, this equals the vertical height *h* multiplied by the cotangent of angle *a*. The cylindrical rods or plugs used for gaging a dovetail by this method should be small enough so that the point *e* of contact is somewhat below the corner or edge of the dovetail.

Estimating Weights of Bar Stock. — The weight of round steel bars per foot of length may be determined by multiplying the diameter of the bar by 4, squaring the product and then dividing the result by 6. This rule expressed as a formula would be:

$$\text{Weight per foot} = \frac{(4 \times d)^2}{6},$$

in which *d* equals the diameter of the stock, in inches. This rule is based on 489 pounds per cubic foot.

Hexagon stock is about 10 per cent heavier than round; square stock, about 28 per cent heavier than round. It is easier, however, to obtain the weight of square or rectangular sections by multiplying the area of the cross-section in square inches by 10, which will give the weight per yard of length. This is based on 480 pounds to the cubic foot, the weight of iron and a unit of length which was in common use when iron was generally used. "Weights per yard," however, have disappeared almost entirely, except in connection with rails, and section books now give the weights of material in pounds per foot. The weight per foot is easily arrived at by taking one-third of the figures obtained by the preceding rule for iron and by adding 2 per cent for weight of steel.

Weight of Sheet Iron per Square Foot. — The weight of sheet iron is found by using the number of thousandths of thickness considered as a whole number and dividing by 25 to obtain the weight in pounds per square foot. Hence $\frac{1}{8}$-inch plate equals 125 thousandths ÷ 25 = 5 pounds. This, too, is based on 480 pounds per square foot and is 2 per cent too light for steel, which would then weigh 5.1 pounds in the above example. The result obtained by using a block 12 inches thick is

$$\frac{12 \times 1000}{25} = 480 \text{ pounds, for iron.}$$

To this add 2 per cent for sheet steel which equals 480 + 9.6 = 489.6 pounds.

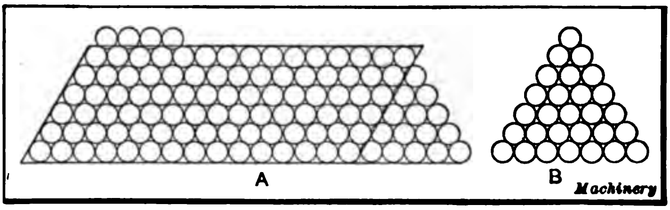


Fig. 8. Diagram representing Bars of Stock in a Pile and illustrating Rapid Method of determining Number

Number of Bars of Stock in a Pile. — When bars of stock, billets or other pieces of uniform size are piled in rows, as illustrated in Fig. 8, the number may be determined without actually counting all the pieces. The method is as follows: Multiply the number of bars or other pieces in the top row by the number of rows, which gives the number contained in the parallelogram shown at *A*. To this result add the number of odd bars, if any, on top of the pile and also the product of one-half the number of rows multiplied by one less than the number of rows in the pile. The latter gives the number of bars shown at the right of the parallelogram. This rule can be stated as a simple formula as follows:

$$N = TS + \frac{S}{2}(S - 1) + O,$$

in which,

N = number of bars;
T = number of bars in top row;
S = number of bars in side row;
O = number of odd bars on top.

For example, in the case shown at A, $T = 15$; $S = 6$; and $O = 4$. Inserting these values in the formula and solving, the number of bars is found to be:

$$15 \times 6 + \tfrac{6}{2} \times (6 - 1) + 4 = 90 + 15 + 4 = 109.$$

Should the pile have only one bar on top, as at B, add 1 to the number of rows, then multiply one-half of this sum by the actual number of rows; or, as a formula:

$$N = S\left(\frac{S + 1}{2}\right).$$

Substituting in this formula the value of S shown at B, or 7, and solving, the number of bars is found to be:

$$7 \times \frac{7 + 1}{2} = 7 \times 4 = 28.$$

Another method for determining the number of bars in a pile without actually counting all of them is as follows: Add the number of bars in the top row to the number of bars in the bottom row and multiply the result by one-half the number of rows. To this result add the odd number, if any, at the top of the pile. This can be stated as a simple formula:

$$S = (T + B)\frac{N}{2} + O,$$

in which,

S = total number of bars;
T = number of bars in top row;
B = number of bars in bottom row;
N = number of complete rows;
O = number of odd bars at top of pile.

For example, in the case shown at A, (Fig. 8) $T = 15$, $B = 20$, $N = 6$, $O = 4$. Inserting these values in the formula and solving:

$$S = (15 + 20)\frac{6}{2} + 4 = 35 \times 3 + 4 = 109.$$

In the case shown at B, $T = 1$, $B = 7$, $N = 7$, $O = 0$. Inserting these values in the formula and solving:

$$S = (1 + 7)\frac{7}{2} + 0 = 8 \times \frac{7}{2} + 0 = 28.$$

To Find Economical Length of Stock for Four-spindle Screw Machine. — Frequently, in multiple-spindle screw-machine work, four bars of unequal length are in the machine and one bar of stock remains to be cut. The problem then arises of cutting this bar into four pieces of such lengths that all the stock will be finished at the same time, yet none of the spindles will be running empty while the others are finishing.

This can be solved by finding the difference in the lengths of the bars in the machine and then finding the amount that must be added to the longest bar. For example, the remaining bar is 8 feet long. The difference between the longest and the next longest bar is 9 inches; between the longest and the third longest bar is 14 inches; and between the longest and the shortest bar is 17 inches.

If x = number of inches to be added to longest bar;
$x + 9$ = number of inches to be added to next longest bar;
$x + 14$ = number of inches to be added to third longest bar;
$x + 17$ = number of inches to be added to shortest bar;
$4x + 40$ = amount to be added to all the bars.

As the remaining bar is 96 inches long, $4x + 40 = 96$; $4x = 96 - 40 = 56$, and $x = 14$. So the lengths into which the bar should be cut are $x = 14$ inches; $x + 9 = 23$ inches; $x + 14 = 28$ inches; and $x + 17 = 31$ inches.

Setting the Sine Bar to a Given Angle. — A simple application of the sine bar is illustrated by the diagram, Fig. 9. A taper plate is to be ground to an angle of 10 degrees 30 minutes. After one edge is finished straight the plate is held against an angle-plate and upon a sine bar, which must be set to the angle required. An accurate sine bar can be set to a given angle within close limits, provided the distance C between the centers of the plugs or bushings attached to the bar is known. Assuming that center distance C equals 10 inches, in this case, then the problem is to find what the vertical distance x between the plugs should be when the sine bar is set to 10 degrees 30 minutes.

Rule: To obtain this distance x, first find the sine of the angle required and multiply this sine by the center distance C.

By referring to a table of sines, we find that the sine of 10 degrees 30 minutes is 0.18223, and distance $x = 0.18223 \times 10 = 1.822$ inch for an angle of 10 degrees 30 minutes.

Setting a Sine-bar Fixture to a Given Angle. — Sine-bar fixtures similar to the one shown in Fig. 10 are sometimes used

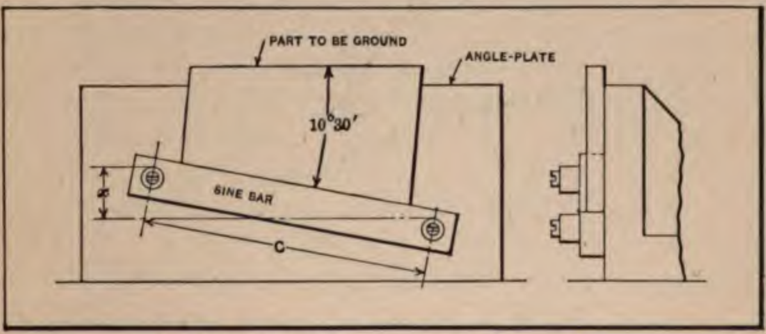


Fig. 9. Simple Application of a Plain Sine Bar

in preference to a plain bar. That part of the fixture which takes the place of the sine bar is in the form of a plate or leaf, and is pivoted at one end to the base. On this fixture, the center of the pivot is exactly $1\frac{1}{2}$ inch from the under side of the base, and the distance from the center of the pivot to the center of the plug on the leaf is 5 inches, as indicated on the illustration. Therefore, the vertical height x from a surface plate on which the fixture is held, to the top of the $\frac{1}{2}$-inch plug on the leaf, is determined for this sine-bar fixture as follows:

Find the sine of the required angle A, multiply the sine by 5 and add to the product 1.5 inch plus the radius of the plug, or $1.5 + 0.25$. Thus, $x = \sin A \times 5 + 1.75$.

Example. — Suppose a gage is to be ground to an angle of 31 degrees. To what distance x should the height gage be set?

$$\text{Sin } 31 \text{ degrees} = 0.51504.$$

Therefore,

$$x = 0.51504 \times 5 + 1.75 = 4.325 \text{ inches.}$$

This fixture has in addition to a plug on the leaf another plug on the base, the center of which is also exactly 5 inches from the center of the pivot. The height x would be required when setting the leaf to a given angle A by means of a height gage or a similar measuring instrument, but if a micrometer

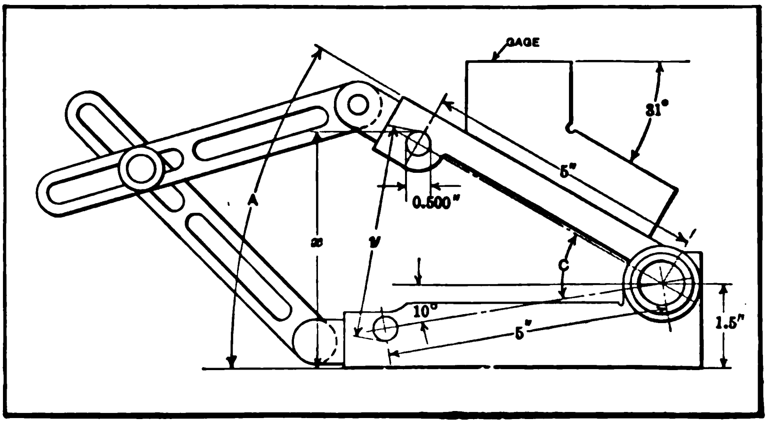


Fig. 10. Sine-bar Fixture

or a vernier caliper were used, the distance y over the plugs would be required. The distance y from the outside of one plug to the outside of the other plug equals the sine of $\frac{1}{2}$ the angle C multiplied by twice the center distance, plus the plug diameter, or $y = \sin \frac{C}{2} \times 10 + 0.5$, for this particular sine bar.

The angle A represents the angle between the leaf and base of the fixture. Angle C for any angle A equals $A + 10$ degrees. The angle of 10 degrees marked on the drawing (Fig. 10) represents the angle between a horizontal line intersecting the axis of the leaf pivot and a line passing through the center of this pivot and the center of the lower measuring plug.

If the sine bar were set for 31 degrees, then angle C would equal $31 + 10 = 41$ degrees, and the sine of $\frac{1}{2}$ angle C or 20 degrees 30 minutes = 0.35021, and $0.35021 \times 10 + 0.5 = 4.002$ inches. The micrometer or vernier caliper would then be set to 4.002 inches.

Measurement of Angles with Sine Bar. — When the sine bar is used for measuring the angle of a finished part, the problem is the reverse of that previously referred to, the object being to determine the angle corresponding to a given measurement instead of the measurement for setting the sine bar to a given angle.

Example. — Assuming that the angle of the tapering plate shown on the sine bar in Fig. 9 is not known, how is this angle determined?

If the top edge of the plate is parallel with the surface plate from which the heights of the plugs are to be measured, the difference x between the heights of the plugs is first determined. This distance is then divided by the center distance C of the sine bar to obtain the sine of the angle. For instance, if x equals 1.822 inch, the sine of the required angle equals $1.822 \div 10 = 0.1822$, which is the sine of 10 degrees 30 minutes.

If both edges of the plate to be measured were at an angle with the surface plate, the angle of each edge would be determined separately by placing the sine bar in contact with first one edge and then the other, and proceeding as just described. The sum of the angles thus obtained would equal the total included angle between the sides.

When using the sine-bar fixture shown in Fig. 10, the sine of angle A for a given measurement x is found as follows: Subtract from height x the radius of the plug plus the distance from the base to the center of the pivot, and divide the remainder by the center distance between the pivot and the sine-bar plug. For this sine-bar fixture, $\sin A = \frac{x - 1.75}{5}$.

Example. — If x equals 4.325 inches, what is angle A?

$\text{Sin } A = \frac{4.325 - 1.75}{5} = 0.515$, which is the sine of 31 degrees.

The angle A for a given measurement y over the plugs is obtained by first finding the sine of one-half angle C which equals $\frac{y - 0.5}{10}$. Angle A is found by subtracting 10 degrees from angle C.

Example. — If y measures 4.002 inches, to what angle A is the sine bar set?

Sine of one-half angle $C = \frac{4.002 - 0.5}{10} = 0.3502$, which is

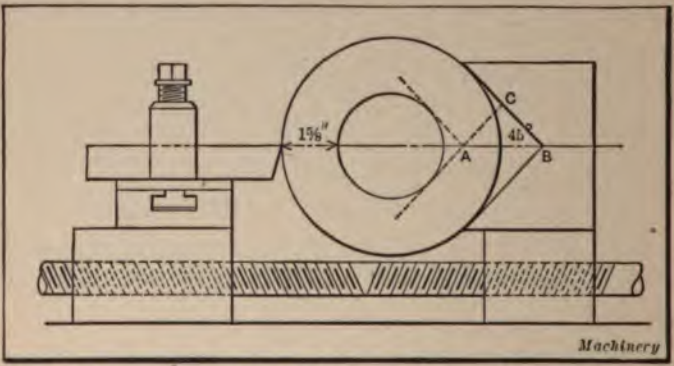


Fig. 11. Diagram for Illustrating Relative Movement between Cutting Tool and V-back-rest

the sine of 20 degrees 30 minutes. Therefore, $C = 41$ degrees, and $A = 41 - 10 = 31$ degrees.

Movement of a Back-rest for Reductions of Diameter. — The diagram, Fig. 11, shows a turning tool and a V-shaped back-rest which are mounted upon slides connected by a right- and left-hand screw. What should be the horizontal movement of a 90-degree V-rest to keep in contact with the work while the tool moves horizontally $1\frac{5}{8}$ inch? If the pitch of the thread that moves the tool is $\frac{1}{8}$ inch, what should it be for moving the rest?

It will be seen that the rest must move horizontally a greater distance than the tool, that is, the horizontal distance AB is greater than the radial distance AC. The distance AB may be calculated as follows:

$AB = AC \div \cos 45$ degrees, or $1\frac{5}{8} \div 0.707 = 2.298$, or $2\frac{19}{64}$ inches.

If a pitch of $\frac{1}{8}$ inch moves the tool, then the pitch for the back-rest must be greater in the following proportion:

$$(2\tfrac{19}{64} \div 1\tfrac{5}{8}) \times \tfrac{1}{8} = 0.177 \text{ or } \tfrac{11}{64} \text{ inch, nearly.}$$

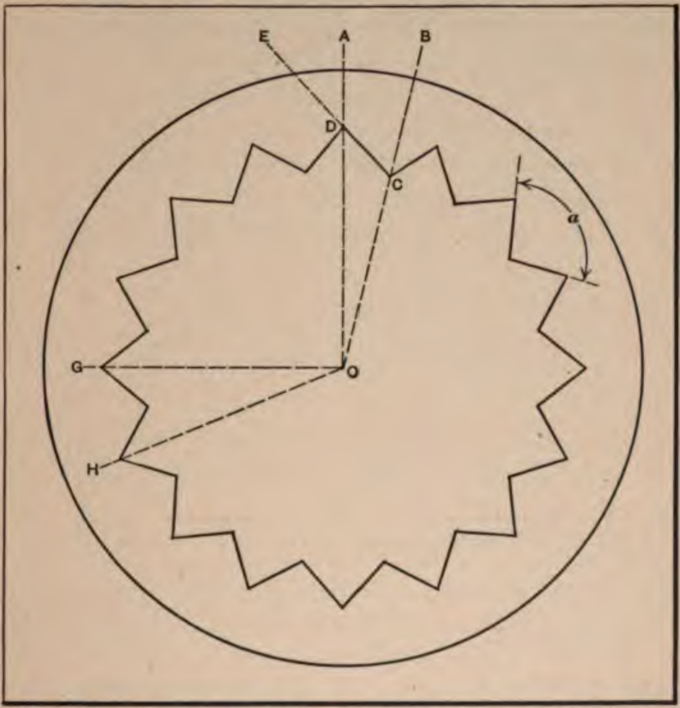


Fig. 12. Ring having V-shaped Notches on Inside, the Problem being to Determine their Angle

A simple arithmetical solution of the problem, because of the angle of the jaws being 90 degrees, may be made as follows:

$$2 \times AC^2 = AB^2, \text{ and}$$

$$AB = \sqrt{2 \times 1\tfrac{5}{8} \times 1\tfrac{5}{8}} = 2.3, \text{ or } 2\tfrac{19}{64} \text{ inches (approximately).}$$

If the jaw angle is not 90 degrees, then the general solution given above may be applied to all cases.

angle to the surface of the machine table. It is also evident that the angle thus obtained may be modified by setting the tool-slide at an angle other than its right-angle position.

If the tool-slide moves horizontally $\frac{1}{4}$ inch for each revolution of the feed-screw, and vertically $\frac{3}{16}$ inch for each revolution of the feed-shaft, the tool will move 4 inches horizontally ($\frac{1}{4} \times 16 = 4$), while it moves vertically 3 inches ($\frac{3}{16} \times 16 = 3$).

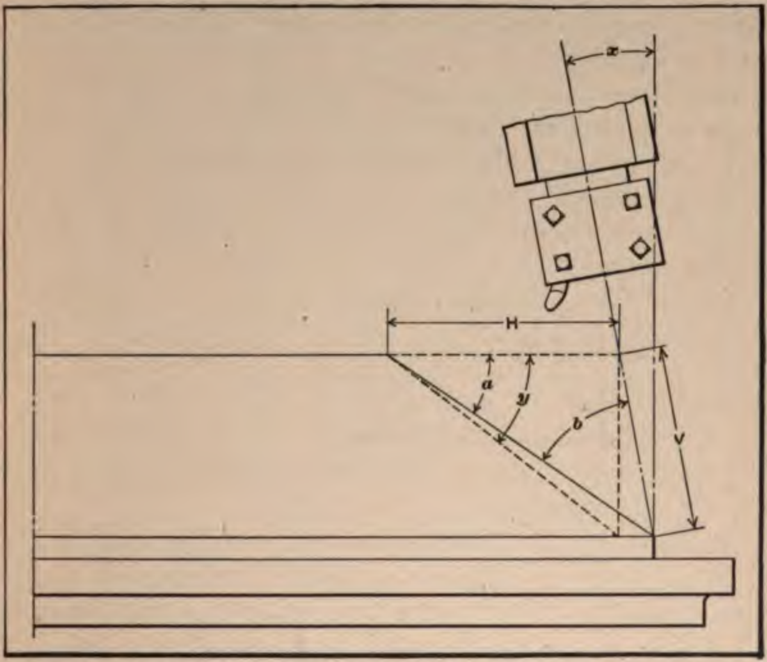


Fig. 13. Tool-slide of Boring Mill set at an Angle for Taper Turning, with Combined Horizontal and Vertical Feeds

If the tool-slide were in the vertical position, the angle obtained by the combined feeds might be either greater or less than the required angle.

Referring to the diagram (Fig. 13) suppose angle y is obtained when the tool-slide is in a vertical position, and that angle a is required. In this case, it will be necessary to set the tool-slide at some angle x so that, as the tool feeds horizontally a distance H, its downward movement V will cause the conical surface to be turned at the required angle a. The first step is to find angle b. To obtain the sine of angle b, multiply the sine of the required angle a by the rate of the horizontal feeding movement, and divide the product by the rate of the vertical feeding movement. Thus, sine b equals $\frac{\sin a \times H}{V}$, in which H and V represent the rates of horizontal and vertical feeding movements, respectively. The difference between the sum of angles a and b and 90 degrees equals the required angle x.

Example. — A conical-shaped casting is to be turned to an angle of 34 degrees as measured from the base of the casting. If the rate of the horizontal feeding movement to the vertical feeding movement is as 4 is to 3, and the combined feeds are used, to what angle x (see Fig. 13) should the tool-slide of the boring mill be set?

The sine of 34 degrees equals 0.559; therefore, the sine of angle b equals $\frac{0.559 \times 4}{3} = 0.7453$, and angle b equals 48 degrees 10 minutes, approximately. The angle x through which the tool-slide is moved from its vertical position equals $90° - (34 + 48° 10') = 7$ degrees 50 minutes, or approximately $10\frac{5}{8}$ degrees.

In this case, the lower end of the tool-slide is moved to the right from its vertical or central position, because the required angle a is less than the angle y obtained from the combined feeds when the tool-bar is vertical. If angle a were greater than y, the lower end of the tool-slide would be moved to the left from its vertical position, and the sum of angles a and b would exceed 90 degrees, so that the latter would be subtracted from this sum.

Taper Turning on Vertical Mill when Housing is set back. — The housing of a vertical boring mill is sometimes set back to permit turning or boring a casting which is too large in diameter to clear the housing when the latter is in its proper position. As an example, assume that a large ring, 14 feet 6 inches in diameter, is to be bored out with a taper of 30 degrees over a surface 6 inches wide, as indicated in the lower left-hand corner of Fig. 14, the diameter of the ring at the point *A* being 14 feet. The only machine available is a 10-foot

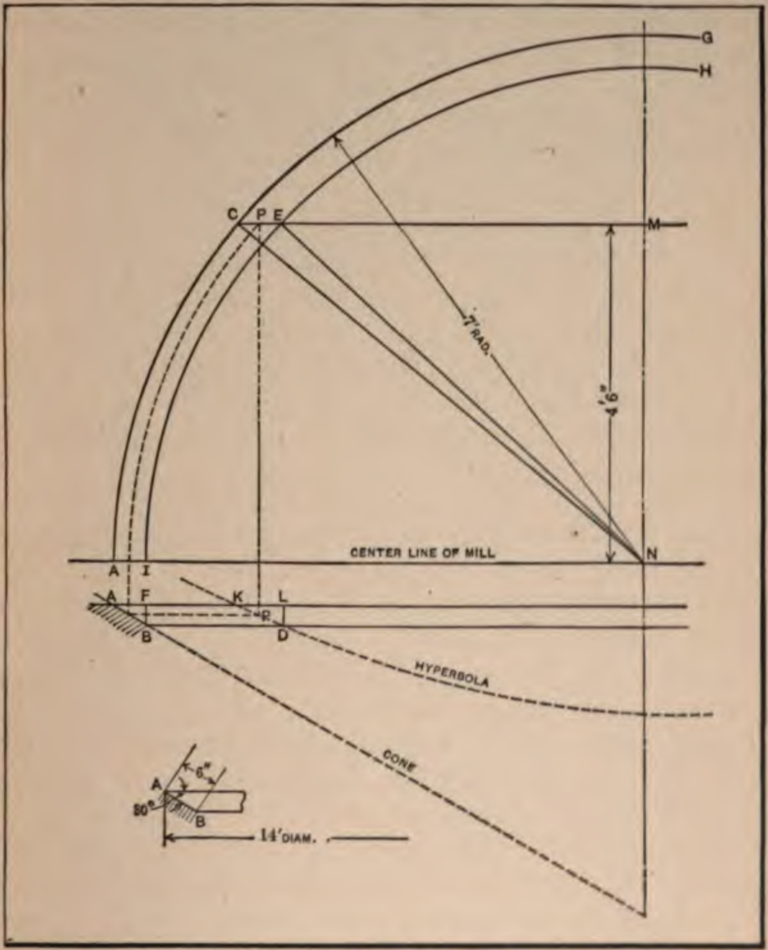


Fig. 14. Illustrating How Angular Position of Tool-slide is determined when Housing is set back for Increasing the Capacity or Swing of the Machine

boring mill, and on account of the table being too small, strips have to be bolted to it to support the ring. The housing must be set back so that the tool is 4 feet 6 inches back from the center line of the mill. The extension head cannot be used, so it is necessary to use the heads on the cross-rail. At what angle must the head be set in this location in order to give the 30-degree bevel?

The angle may be obtained either from a carefully drawn diagram or by a simple calculation. It should be noted, however, that feeding the tool in a straight line under the conditions given will not produce a perfectly straight beveled surface, but one that is slightly convex, although the inaccuracy in this case is slight.

Considering first the graphical method, draw, preferably to a large scale, a sectional view and plan of the bevel ring, as shown in the illustration. Line *CE* represents the path of the tool. Points *C* and *E* projected to *K* and *D* give line *KD*, which also represents the path of the tool. Angle *DKL*, therefore, is the angle to which the head should be set. This angle can be measured by a protractor, if the drawing has been carefully made. It will, in the given case, be found to equal $23\frac{1}{2}$ degrees.

In order to ascertain if the line *KD* corresponds fairly well with the hyperbola along which the tool should properly be fed so as to produce a perfectly straight face at *AB*, construct the curve which forms the intersection between plane *CM* and the cone of which beveled face *AB* is a part. The construction of the intersecting curve, which is one of the problems found in practically all text-books on mechanical drawing, is accomplished by projecting points *P* from the plan view to the sectional view as indicated. It will be found that in the present case the hyperbola almost coincides with the straight line between points *D* and *K*. Hence, the inaccuracy produced, that is, the convexity of face *AB* will be slight.

To calculate angle *DKL*, proceed as follows: First find the length of *CM*.

$$\sqrt{CN^2 - NM^2} = CM. \text{ Hence, } CM = \sqrt{84^2 - 54^2} = 64.34 \text{ inches.}$$

Next find the length of *EN* and of *EM*.

$$EN = CN - AF = 84 - 6 \times \cos 30 \text{ deg.} = 78.8 \text{ inches.}$$

$$EM = \sqrt{EN^2 - NM^2} = \sqrt{78.8^2 - 54^2} = 57.39 \text{ inches.}$$

Then, $CE = CM - EM = 6.95 = KL$.

$$KL \div LD = \cot DKL.$$

But $LD = AB \times \sin 30$ deg. $= 3$ inches.

Hence, $\cot DKL = 6.95 \div 3 = 2.317$, and $DKL = 23\frac{1}{3}$ degrees, which is nearly the same as the angle of $23\frac{1}{2}$ degrees obtained by the graphical solution.

Travel of Cutter when Milling Gear Teeth. — It is well known that in milling gear blanks a certain allowance must be made before the cutter cuts to the full depth, and that this must be added to the face width when figuring the cutting time. The diagram (Fig. 15) shows this extra travel which is represented by the letter x. This amount must be added to the face of the gear to obtain the full travel required.

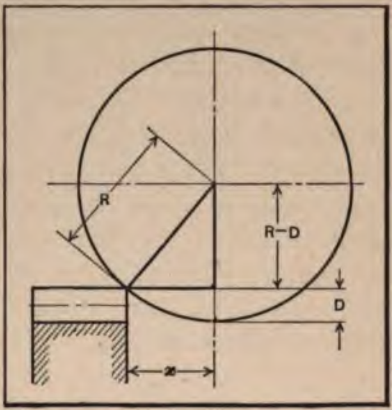


Fig. 15. Diagram illustrating Travel of Cutter when Milling Gear Teeth

The method of figuring this extra travel requires only the solution of a right-angle triangle in which the hypotenuse R represents the cutter radius; $R - D$, the perpendicular; x, the base of the triangle; D, the whole depth of the tooth. In the example given, the cutter radius is 1.5 inch. For a five-pitch cutter, $D = 0.4314$; therefore, $R - D = 1.5 - 0.4314 = 1.068$; and $x = \sqrt{1.5^2 - 1.068^2} = \sqrt{2.25 - 1.14} = 1$ inch, approximately, which should be added to the face width of the gear when estimating the time required for milling the teeth.

Proportioning Gears when the Center Distance and the Number of Teeth are fixed. — Suppose the center-to-center distance between two shafts is fixed and it is desired to use gears of a certain pitch; how can the number of teeth in each gear for a given speed be determined?

Since the gears must be of a certain pitch, the total number of teeth available may be determined and then the number of teeth in the driving and driven gears. The total number of teeth equals twice the product of the center distance multiplied by the diametral pitch. If the center distance is 6 inches and the diametral pitch, 10, the total number of teeth equals $6 \times 10 \times 2 = 120$ teeth. The next step is to find the number of teeth in the driving and driven gears for a given rate of speed.

Rule: Divide the speed of the driving gear in revolutions per minute by the speed of the driven gear and add one to the quotient. Next divide the total number of teeth in both gears by the sum previously obtained, and the quotient will equal the number of teeth in the driving gear. This number subtracted from the total number of teeth will equal the number of teeth required in the driven gear.

Example. — If the center-to-center distance is 6 inches, the diametral pitch, 10, the total number of teeth available will be 120, as previously explained. If the speeds of the driving and the driven gears are to be 100 and 60 revolutions per minute, respectively, find the number of teeth for each gear.

$$\frac{100}{60} = 1\tfrac{2}{3} \text{ and } 1\tfrac{2}{3} + 1 = 2\tfrac{2}{3}.$$

$$120 \div 2\tfrac{2}{3} = \frac{120}{1} \times \frac{3}{8} = 45 = \text{number of teeth in driving gear.}$$

The number of teeth in the driven gear equals $120 - 45 = 75$ teeth.

The following formula may also be used for solving problems of this kind:

$$N = \frac{C2RP}{R + R_1} \qquad n = \frac{C2R_1P}{R + R_1}.$$

In this formula,

C = center-to-center distance between gears;
R and R_1 = terms of the ratio (substitute highest term for R);
P = diametral pitch;
N = number of teeth in large gear;
n = number of teeth in small gear.

Example. — If this formula is applied to the preceding example, the terms of the ratio equal $\frac{60}{100} = \frac{3}{5}$. Then,

$$N = \frac{6 \times 2 \times 5 \times 10}{5 + 3} = 75;$$

$$n = \frac{6 \times 2 \times 3 \times 10}{5 + 3} = 45.$$

The larger gear is placed on the shaft which is to run at the slowest speed. In this case, the driven shaft runs at the slowest speed; hence, the driven gear has 75 teeth.

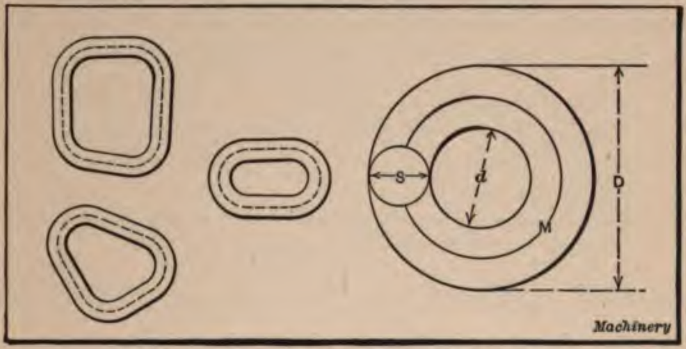


Fig. 16. Rings of Odd and Circular Shapes

When the center distance and velocity ratios are fixed by some essential construction of a machine, it is usually impossible to use standard diametral pitch gear teeth. If cast gears are to be used, it does not matter so much, as a patternmaker can lay out teeth according to the pitch desired, but if cut gears are required, an effort should be made to alter the center distance so that standard cutters can be used.

Mean Circumference of a Ring. — To obtain the mean circumference of a ring, divide the sum of the outside and inside diameters by 2, and multiply 3.1416 by the quotient.

In Fig. 16, D equals the outside diameter, d equals the inside diameter, and M equals the mean circumference. Then $M = 3.1416 \times \frac{D + d}{2}$.

Example. — D = 10 inches; d = 8 inches. What is the mean circumference? $M = 3.1416 \times \frac{10 + 8}{2} = 28.27$ inches.

When rings are of odd shape or are not circular, the mean circumference may be obtained if the diameter of the stock and the length of the periphery, either on the inside or outside, are known. If the measurement around which the part is to fit is known, this measurement is considered as a circumference and is divided by 3.1416 to obtain a corresponding diameter. To this diameter, add the diameter of the stock and multiply the sum by 3.1416, thus obtaining the

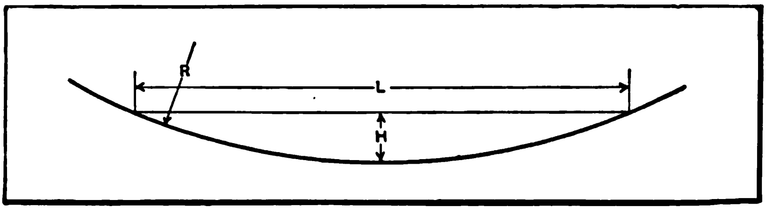


Fig. 17. Method of determining Radius of Large Curve

length of the odd-shaped ring or what corresponds to the mean circumference.

Example. — Suppose a ring similar to one of the shapes shown at the left-hand side of Fig. 16 is made of 1-inch stock and is to fit over a part which measures $25\frac{1}{8}$ inches around, the measurement being taken by means of a flexible steel tape. The diameter corresponding to $25\frac{1}{8}$ inches, or $25.125 \div 3.1416$, equals 8 inches, approximately; hence, the mean circumference or length of the stock equals $3.1416 \times (8 + 1) = 28\frac{1}{4}$ inches, nearly.

Radius of Large Curves. — It is sometimes necessary to find the radius of a large curve, the center of which is not accessible. For instance, the curved part may be a circular plate or a templet for an arc. A common method of determining the radius is indicated by the diagram (Fig. 17). A straightedge of any convenient length L is placed on the curved part, and the height H is measured, this height being the distance between the straightedge and the middle part of the curve. One method of determining the radius is to lay out the chordal distance L and the height H on drawing paper to any convenient scale and then find the radius by trial, and measure it directly. A more accurate method is to calculate the radius.

Rule: Add the square of one-half the length of the straightedge to the square of the height between the straightedge and the curved part; then divide this sum by twice the height between the straightedge and the curved part. The quotient thus obtained will equal the radius of the curve. This rule expressed as a formula is as follows:

$$R = \frac{H^2 + l^2}{2\,H};$$

in which,

R = radius of the curved arc;
H = height of the arc;
l = one-half the chord, or one-half the length of the straightedge.

Radius of a Circumscribed Circle. — The three smaller circles in the diagram, Fig. 18, represent three hardened and ground plugs, and the problem is to determine the radius R of a ring which will be tangent to all three of the plugs. The only dimensions known are distances P and C between the center lines of the plugs. To solve this problem, the radii of the three plugs are first required. The radius b of each of the two smaller plugs equals one-half of C. The radius a of the larger plug must be calculated. It will be seen that $a = mn - b$, but $mn = \sqrt{mh^2 + hn^2} = \sqrt{p^2 + (\frac{1}{2}\,C)^2}$. Having thus determined the radius a, the radius R should be determined, which is the quantity to be ultimately found in the problem.

Assume that the center of the large circle to be found is at o. The length om, which is not known, is called x. Two equations can now be written, which can be simplified so as to contain only the two unknown quantities x and R. As the first equation we have:

$$R = om + ml = x + a.$$

As the second equation:

$$R = on + nk = \sqrt{oh^2 + hn^2} + nk = \sqrt{(P - x)^2 + b^2} + b.$$

As the members on the right-hand side in both of these equations equal R, they are also equal to each other. Thus:

$$x + a = \sqrt{(P - x)^2 + b^2} + b.$$

If this equation is solved for x, then,

$$x = \frac{P^2 - a^2 + 2\,ab}{2\,a - 2\,b + 2\,P},$$

and

$$R = a + \frac{P^2 - a^2 + 2\,ab}{2\,a - 2\,b + 2\,P}.$$

Example. — If the vertical distance P (see Fig. 18) between the center lines equals 1.2 inch, and the distance C between the centers of the small plugs equals one inch, what is the radius R of a circumscribed circle?

The radius $b = \frac{1}{2}$ the center distance C, or 0.5 inch. Radius $a = \sqrt{1.2^2 + 0.5^2} - 0.5 = \sqrt{1.69} - 0.5 = 1.3 - 0.5 = 0.8$. If the values of P, a, and b are inserted in the expression previously given for R, then,

$$R = 0.8 + \frac{1.2^2 - 0.8^2 + 2 \times 0.8 \times 0.5}{2 \times 0.8 - 2 \times 0.5 + 2 \times 1.2} = 1.333 \text{ inch.}$$

The radius of a circumscribed circle may also be determined by another method. The radius a is first calculated, as previously described. If the distance ld is equal to the radius b, then dh equals $mh + md$, or $P + md$; and md equals the difference between lm and b. The tangent of angle hdn equals $\frac{hn}{dh}$. The angle hon equals 2 times angle hdn, and on equals cosecant angle hon times hn. Then, $R = on + nk$.

Average Speeds when Periods of Time Vary. — When determining average speeds, the arithmetical mean between two different rates of speed does not equal the average speed if the periods of time vary. If a railway train runs to a point 50 miles distant and back to a given point at a speed of 40 miles per hour, and another train makes the same trip at a speed of 35 miles per hour on the outward trip and 45 miles per hour on the return trip, the average speed of the two trains will not be the same. The train running at the uniform speed of 40 miles per hour will arrive first at the starting point, because its average speed is higher. The time required on

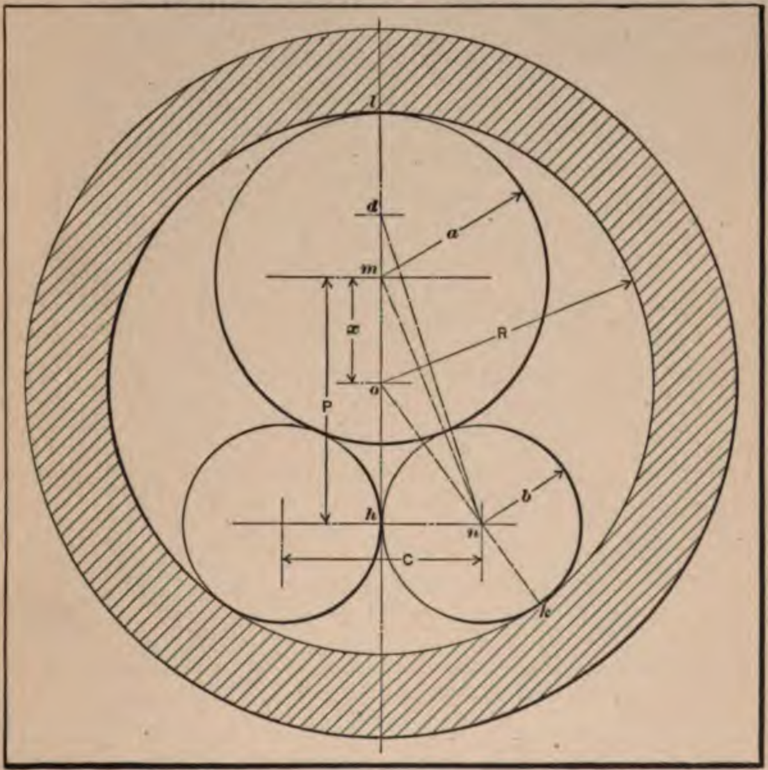


Fig. 18. Values required in Determining Radius *R* of Ring containing Three Close-fitting Plugs

the outward trip by the 35-mile per hour train is greater than the time required when returning at 45 miles per hour; consequently, it is going at the slower rate longer than at the higher rate of speed. The result is that the average speed for the outward and return trips is not the arithmetical mean between 45 and 35. The average speed equals: $\frac{35\,T + 45\,T_1}{T + T_1}$, assuming that T = the number of hours required for the outward trip; T_1 = the number of hours required for the return trip. In the example referred to, the time required for the outward trip of the train running at different rates of speed is equal to 50 ÷ 35 = 1.428 hour, and the time required for the return trip equals 50 ÷ 45 = 1.111 hour; hence,

$$\text{Average speed} = \frac{35 \times 1.428 + 45 \times 1.111}{1.428 + 1.111} = \frac{99.97}{2.539} = 39.37$$

miles per hour.

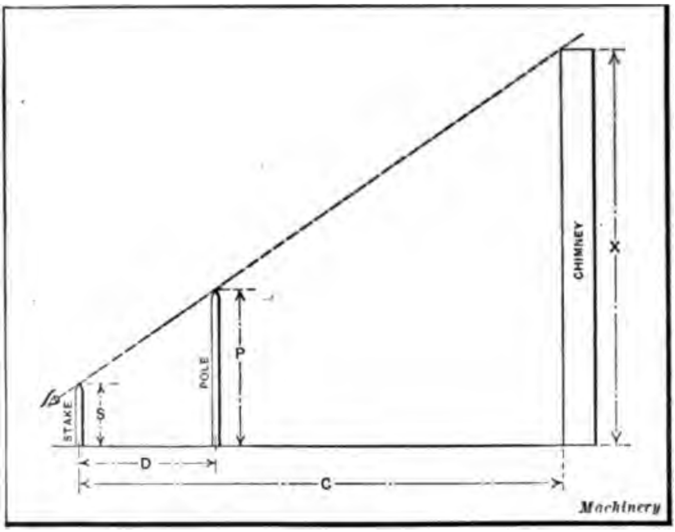


Fig. 19. Method of finding the Height of a Stack or Chimney

Average Speed of a Planer Table. — There is a common application in ordinary shop work of the principle involved in this problem. A planer has a cutting speed of 20 feet per minute, and a return speed of 60 feet per minute. At first thought, it may seem that the average speed of the planer platen is 40 feet per minute, but that conclusion is not correct. For simplicity, assume the exaggerated condition in which the stroke of the planer is 60 feet. The cutting speed being 20 feet per minute, the forward stroke will require 3 minutes; and the return speed being 60 feet per minute, the return stroke will require one minute. The total time required for one forward and one return stroke is thus 4 minutes. During this time the platen has traveled two times the stroke, or 120 feet, and the average speed is 30 feet per minute. The formula for finding the average speed could be expressed:

$$\frac{2S}{\frac{S}{C}+\frac{S}{R}} = \text{average speed per stroke},$$

in which S = length of the stroke in feet;
C = cutting speed in feet per minute;
R = return speed in feet per minute.

This formula can be simplified so as to take the form:

$$\frac{2CR}{R+C} = \text{average speed per stroke}.$$

If we substitute in this formula the figures of the planer problem previously given, then:

$$\frac{2 \times 20 \times 60}{20 + 60} = \frac{2400}{80} = 30.$$

It should always be borne in mind that the average speed is the arithmetical mean between two given speeds only when the periods of time during which each speed is in operation are equal. In this case of forward and return strokes at different speeds, one stroke is made in a shorter time than the other, and the average speed is not expressed by the arithmetical mean of the two speeds.

Height of a Chimney or Stack. — To determine the height of a chimney or stack, select the most convenient place in sight of the chimney top and erect a short pole, as shown in Fig. 19. At a convenient distance from the pole, say eight or ten feet, drive a stake in line with the pole and the chimney; the tops of the stake and the pole may be bluntly pointed. See that both the stake and the pole are plumb. Drive the stake into the ground until the tops of the stake and pole are in line with the top of the chimney, or, if more convenient, mark the stake at the point that is in alignment with the pole top and the chimney top. Mark a ground line on the stake level with the ground line of the pole. Measure the distance C from the chimney to the stake, the distance D from the stake to the pole, the height P of the pole, and the height S of the stake. Then the height X of the chimney is:

$$X = \frac{C}{D}(P - S) + S.$$

For the formula as given, the ground line at the pole and stake must be level with the base of the chimney. If the base of the pole is above or below the base of the chimney, add or subtract this difference from the calculated height X. If C is 160 feet, D is 8 feet, P is 7 feet, and S is 2 feet 4 inches, then, substituting the values and reducing all measurements to inches:

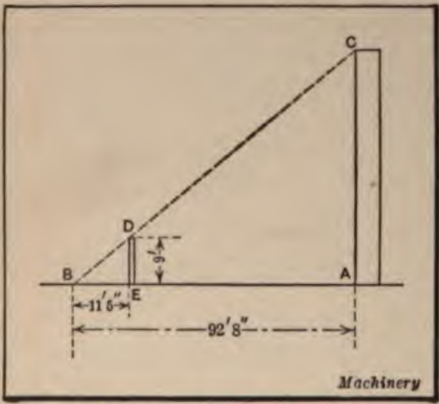


Fig. 20. Another Method of calculating the Height of a Stack or Chimney

$$X = \frac{1920}{96} \times (84 - 28) + 28 = 1148 \text{ inches} = 95 \text{ feet } 8 \text{ inches.}$$

If the ground line at the pole is 2 feet above the base of the chimney, the total height of the chimney will be 95 feet 8 inches plus 2 feet, or 97 feet 8 inches.

This method will give accurate results that can be verified at any time; it can also be varied to suit conditions. For instance, a mark on the corner of a building will often serve in place of the pole. The measurements may be made on a sloping roof if necessary, taking into account all differences of level.

The height of a chimney or other tall structure may also be found by using an engineer's transit. Set up a transit at

B (Fig. 20), measure very carefully the distance *BA*, sight the telescope to the top of the chimney, and measure the angle *CBA*. Then, in the right triangle *CBA*, the side *AB* and the angle *B* are known, from which the height *AC* can readily be found by multiplying the distance *AB* by the tangent of the angle.

The height can also be determined quite closely, in the following manner: Select a time when the sun is about midway between the horizon and the zenith, so that it will cast a comparatively long shadow. Take a pole of some convenient length, the longer the better, stand it upright so that the end of its shadow will just reach to the end of the shadow cast by the chimney, and measure the distance from the pole to the end of its shadow, which will be the same as the distance *EB*. Also measure the length of the shadow cast by the chimney, which corresponds to the distance *AB*. Then, from the similar triangles *ACB* and *EDB*, $AC(=x) : ED = AB : EB$. Suppose $AB = 92$ feet 8 inches, $EB = 11$ feet 5 inches, and the pole is 9 feet long; then $x : 9 = 92\frac{8}{12} : 11\frac{5}{12}$, or $x : 9 = 1112 : 137$; from which $x = 73$ feet, very nearly, which is the height of the chimney.

CHAPTER XVI

EXAMPLES IN ELEMENTARY MECHANICS

The problems in this chapter are not directly applicable to machine-shop or tool-room practice, but illustrate important principles which should be understood by all who desire a broader mechanical training than can be acquired from shop experience alone. When the student of mechanics understands the fundamental principles or what might be defined as the "foundation principles," many problems are simplified which otherwise would prove difficult. Such knowledge is of especial value to those who attempt to originate new mechanical devices, whether as inventors or designers. Many inventions are worthless because the inventor did not understand the first principles of mechanics. The numerous attempts to develop perpetual motion machines are notable examples of waste effort resulting from a lack of elementary mechanical knowledge.

Work and Power. — When a force causes some body, such as a machine part, to move in opposition to a resistance, this is known in mechanics as "work," and it is the result of force and motion. If the force is not great enough to overcome the resistance, no motion occurs and no work is done, according to the use of this term in mechanics. When one pound is raised vertically one foot against the resistance of gravity, the work done is equivalent to one foot-pound, and this is a unit of work. If a casting weighing 100 pounds is lifted to a bench 3 feet high, the work done is equivalent to 300 foot-pounds. It is evident, then, that if F = foot-pounds of work, W = weight in pounds, and H = height in feet, then the number of foot-pounds of work is $F = W \times H$.

It should be noted that the time is not considered in determining the total amount of work done. If 100 pounds are lifted vertically 3 feet, the work done is 300 foot-pounds, regardless of whether the time required for lifting the weight is, say, two seconds, five minutes, or any other period of time, but the amount of *power* required to lift a weight or to do other work is dependent upon the time element. For instance, if 300 foot-pounds of work must be done in two seconds and an electric motor is to be used for doing the work, a larger and more powerful motor is required to perform the work in two seconds than would be needed if the period of time were extended to, say, five minutes, because when the weight is raised very slowly, the rotary motion of the motor is transmitted through a hoisting mechanism which is designed to give a large reduction of motion; hence, a smaller motor may be used. This point is further illustrated by the simple fact that one man can raise heavy machines or other bodies by using a jack, but the rate at which the object is elevated is very slow. If the design of the jack were changed so as to increase the speed of elevation, more power would be required to operate it. In this connection, it is important to remember that whenever any form of mechanism is changed so that it is capable of exerting greater power without increasing the force at the driving end, there is always a corresponding reduction in the speed or rate at which the work is done. The term "power" as used in mechanics should not be confused with force.

Meaning of the Term "Horsepower." — The amount of power in foot-pounds per minute required for doing a certain amount of work is equal to the work in foot-pounds divided by the time in minutes required to do the work. For instance, if 2000 pounds are raised 30 feet in two minutes, the power in foot-pounds per minute equals $\frac{2000 \times 30}{2} = 30{,}000$ foot-pounds per minute. The amount of power in this case would be reduced one-half, or to 15,000 foot-pounds per minute, if the time were increased to four minutes.

One horsepower is equivalent to 33,000 foot-pounds per minute, and this is the unit commonly used to indicate the power of steam engines, gas engines, etc. If a gas engine has a rating of one horsepower, this means that it is capable of performing 33,000 foot-pounds of work per minute, although such an engine may have a maximum power which is somewhat greater than its rated power.

Example. — How many horsepower will be required to raise 8400 pounds 20 feet in two minutes, assuming that all frictional resistance is neglected?

The foot-pounds of work equal $8400 \times 20 = 168{,}000$; therefore, the number of horsepower equals $\frac{168{,}000}{2 \times 33{,}000} = 2.54$ horsepower, approximately.

The Principle of Work. — One of the most important principles in mechanics is known as the "principle of work." According to the principle of work, the amount of work or energy put into a machine equals the work done by that machine plus the energy that is lost. This law or principle holds true for all classes of mechanical apparatus from a simple lever to the most complicated mechanism. In every form of mechanism there are some losses. For instance, it is not possible to eliminate friction entirely, although it may be greatly reduced by the use of proper lubricants, and especially by using ball bearings whenever it is practicable to employ them. The work that is put into a machine is represented by the product of the force and the distance through which it moves. The force may be derived from the pressure of steam, from the pull of a belt, or in some other way.

As the force is transmitted to the point where work is to be done, it may be applied in various directions and the amount changed according to the arrangement of the mechanism and the friction between the different moving parts, which must be overcome. For instance, the rotation of a pulley or flywheel may be changed through a crank and connecting rod into a rectilinear motion, as in the case of a punch press. Finally the modified force is utilized in performing a certain operation or in doing useful work. This means that resistance *must be* overcome, as, for example, the resistance of metal to the cutting and bending action of a blanking and drawing die. Now, according to the principle of work, if frictional or other losses are neglected, the applied or original force multiplied by the distance through which it moves, equals the resistance overcome multiplied by the distance through which it is overcome. Therefore, the work put into a machine, or the energy required for driving it, equals the lost energy plus the work done by that machine. If this simple principle were universally understood, there would be no attempts to originate a perpetual motion machine.

Principle of Work applied to Wheel and Axle. — The well-known wheel and axle is illustrated by the diagram, Fig. 1,

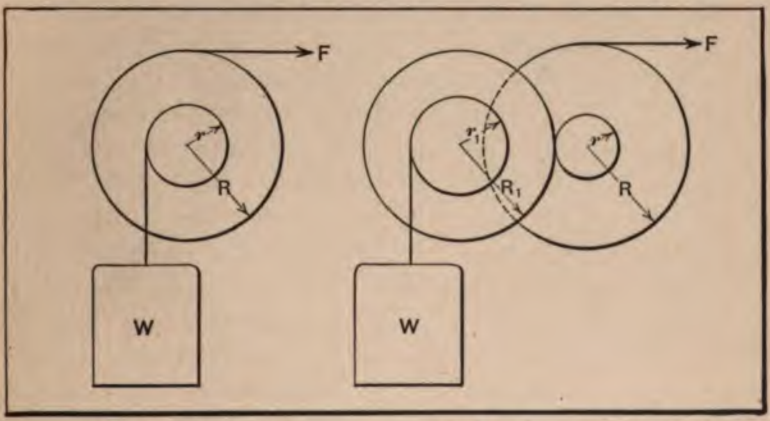


Fig. 1. Wheel and Axle **Fig. 2. Compound Gear Train**

and it will be used as a simple example to illustrate the principle of work. The weight W is suspended at the end of a rope which is wound about a cylindrical drum of radius r and another rope is wound about the larger drum of radius R. As radius R is larger than radius r, the force F required to raise the weight W can be proportionately smaller; thus, $F : W :: r : R$. Therefore,

$$F = \frac{W \times r}{R}, \text{ and } W = \frac{F \times R}{r}.$$

Example. — If a weight W of 500 pounds is to be raised, how much force F will be required if $R = 10$ inches, $r = 3$ inches, neglecting the friction of the wheel and axle bearings?

The force F in pounds required to raise 500 pounds equals $\frac{500 \times 3}{10} = 150$ pounds. If the larger drum were replaced by a crank, the preceding formulas would still apply, the length of the crank being represented by radius R.

Example. — The diagram, Fig. 3, shows a flywheel with an axle 2 inches in diameter; as the flywheel turns, it winds up a rope on the axle, and thus raises a weight as shown. If the energy of the flywheel is equivalent to 5000 foot-pounds, and it will raise a weight of 2500 pounds 2 feet when the diameter of the axle is 2 inches, will it raise the same weight 4 feet if the diameter of the axle is 1 inch?

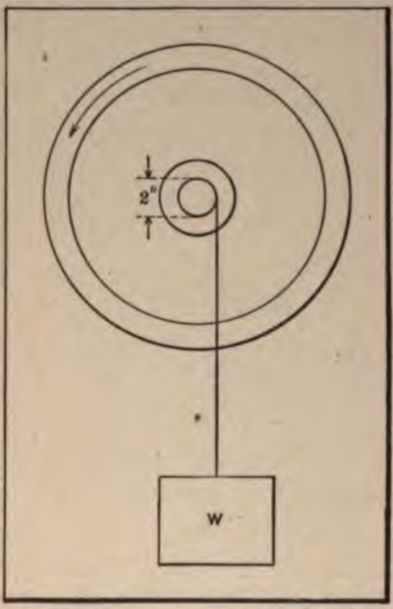


Fig. 3. Diagram illustrating Principle of Work

The flywheel will raise a weight of 2500 pounds 2 feet regardless of the diameter of the axle or drum which has nothing to do with the case. According to the law of the principle of work, the energy of the flywheel is equal to the work expended in enabling it to store up this energy; hence, the force multiplied by the distance through which it moves equals 5000 foot-pounds, equals the resistance (weight) multiplied by the distance through which it is overcome, equals 2500×2. It will thus be seen that it does not matter how the load is raised (neglecting friction and other resistances); all that need be considered is the number of pounds that the load weighs and the height through which it is raised.

Weight Lifted by Compound Wheel and Axle. — The sketch of a hoisting device known as a "Chinese windlass" or a compound wheel and axle is shown in Fig. 4. Suppose that the diameter of the wheel A is 42 inches, and that the diameters of the drums B and C are 10 inches and $8\frac{1}{2}$ inches, respectively; how large a weight can be lifted if a force of 40 pounds is applied at the circumference of the wheel?

The principle of work applies here as in the case of any other machine; i.e., the force multiplied by the distance through which it moves equals the resistance overcome multiplied by the distance through which it is overcome, neglecting frictional losses. Let R, r, and r' be the radii of A, B, and C, respectively, and suppose A to make one revolution; then the distance moved by the force will equal the circumference of wheel A, or $2\pi R$. At the same time, B and C make one revolution also. As the part P of the rope winds on drum B an amount equal to $2\pi r$ and rope Q winds off drum C an amount equal to $2\pi r'$, the rope is shortened an amount equal to $2\pi r - 2\pi r' = 2\pi(r - r')$. The weight W representing the resistance is raised only one-half this distance, however, since it is divided equally between P and Q, D being a movable pulley. Hence, the distance W moves is $2\pi(r - r') \div 2 = \pi(r - r')$. Therefore, if F represents the force applied at the circumference of the wheel A, then $F \times 2\pi R = \pi(r - r') \times W$; from which $W = \frac{2FR}{r - r'}$.

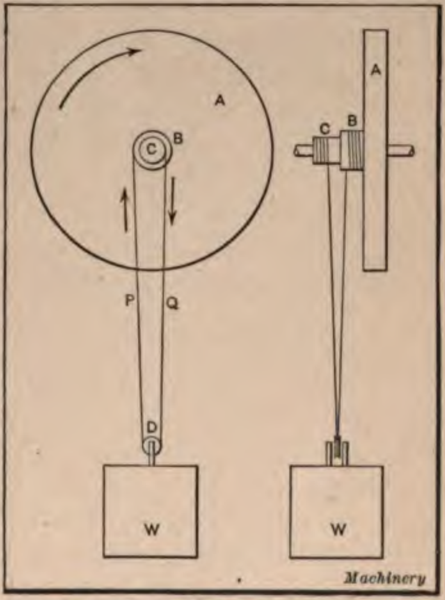


Fig. 4. Compound Wheel and Axle or Chinese Windlass

Substituting the values given in this formula,

$$W = \frac{2 \times 40 \times 21}{5 - 4\frac{1}{4}} = 2240 \text{ pounds.}$$

A mechanism of this kind ought to have an efficiency of at least 0.90; hence, the weight lifted ought to be at least $2240 \times 0.90 = 2016$ pounds.

When the Force is transmitted through a Gear or Pulley Train. — The principle of a simple wheel and axle may be applied to a train of mechanism, such as a train of pulleys or of gearing. In a train of this kind, the continued product of the applied force and the radii of the *driven* wheels equals the continued product of the resistance and the radii of the *driving* wheels. In a wheel and axle (see diagram, Fig. 1), the axle is really a driver and the wheel is driven. Now, if F represents the force as before; W, the weight or resistance; R and R_1 the radii of the driven gears; and r and r_1 the radii of the driving gears, as shown by the diagram, Fig. 2; then,

$$F \times R \times R_1 = W \times r \times r_1.$$

Therefore,

$$F = \frac{W \times r \times r_1}{R \times R_1}, \text{ and } W = \frac{F \times R \times R_1}{r \times r_1}.$$

Example. — If the pitch diameters of the gears shown in Fig. 2 are such that radius $R = 6$ inches, $r = 2$ inches, $R_1 = 5$ inches, and $r_1 = 2\frac{1}{2}$ inches, and a force F of 500 pounds is applied, what weight W can be lifted if the loss of energy from friction is neglected?

$$W = \frac{500 \times 6 \times 5}{2 \times 2.5} = 3000 \text{ pounds.}$$

When Force is transmitted through Pulley Combinations. — The principle of work is further illustrated by the diagrams, Fig. 5, which show two different combinations of pulleys. The problem in this case is to determine the amount of force P that will be required to raise weight W. The pulleys shown by the right-hand diagram will be considered first. In this diagram, A is a movable pulley and B a fixed pulley. If the pulley A be lifted upward through a distance s, part a

of the rope must be shortened an amount s and part b must be shortened the same amount, in order to keep the rope in contact with the pulley; in other words, a point or mark on the rope b will move upward a distance $2s$. Pulley B evidently exerts no influence other than to change the direction of the power from upward to downward. Hence, while W moves through a distance s, P moves through a distance $2s$. Now, according to the principle of work, $Ws = P \times 2s$, or $P = \frac{1}{2} W$, for the case of one movable pulley. Referring now to the left-hand diagram, if W be lifted a distance s, then, since pulley A is fixed, pulley B will descend a distance s; and since pulley B is movable, pulley C will descend $2s$ in consequence of the descent of pulley B, and it will also descend an additional distance s by reason of the ascent of W through that distance. In other words, C descends $2s + s$. Similarly, D descends twice as far as C and through an additional distance s, or $2(2s + s) + s = 4s + 2s + s = (2^2 + 2 + 1)s$. The free end of the rope h descends twice as far as pulley D and an additional distance s, or $2(2^2 + 2 + 1)s + s = (2^3 + 2^2 + 2 + 1)s = 15s$. Consequently, when W moves through a distance s, P moves through a distance $15s$; hence, $P \times 15s = Ws$, or $P = \frac{1}{15} W$. If $W = 1200$ pounds, $P = 1200 \div 15 = 80$ pounds, neglecting friction.

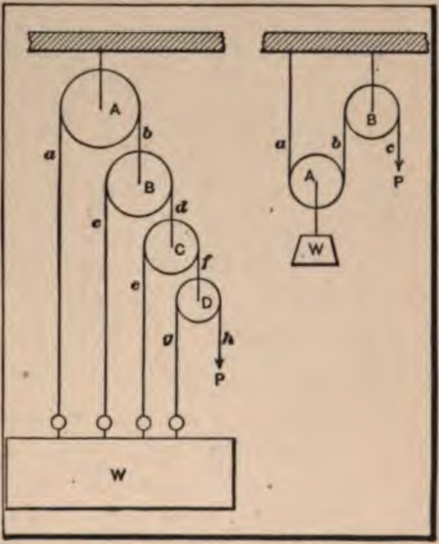


Fig. 5. Pulley Combinations

Energy Resulting from Motion. — When a body has the capacity of doing work or overcoming resistance it is said to possess *energy*. The energy is practically stored up in such a body, and it may be kinetic or potential energy. The work which a flywheel in motion is capable of doing is an example of kinetic energy, the latter being the energy resulting from a body in motion. Potential or latent energy is the capacity for doing work possessed by a body on account of its condition or position. For example, a weight that has been lifted to some point possesses potential energy, and when the weight falls, this potential energy is changed to kinetic energy. Water stored in a reservoir is another example of potential energy.

If a cast-iron "skull-cracker," such as is used for breaking up castings, weighs 300 pounds, and is suspended 16 feet from the ground, it possesses 4800 foot-pounds of potential energy, because, when the weight was raised 16 feet, 4800 foot-pounds of work were expended. When the "skull-cracker" is released and it strikes a casting on the ground, the kinetic energy expended is equal to 4800 foot-pounds. Energy is acquired by a body as the result of work done upon it, as when a flywheel is set in motion or when water is pumped up into a reservoir. If E = energy in foot-pounds; V = velocity in feet per second; W = weight; then,

$$E = \frac{1}{2} \times \frac{WV^2}{32.16} = \frac{WV^2}{64.32}.$$

Example. — If the head of a steam-hammer weighs 800 pounds and it is moved at the rate of 30 feet per second, at the instant it strikes a steel block, what is the kinetic energy?

$$E = \frac{800 \times 30^2}{64.32} = 11,194 \text{ foot-pounds.}$$

Example. — If a body weighs 200 pounds and moves at the rate of 40 feet per second, what is its kinetic energy, and how many horsepower would be required to give it this amount of kinetic energy in five seconds?

$$E = \frac{200 \times 40^2}{64.32} = 4975 \text{ foot-pounds.}$$

The next step is to determine the number of horsepower required to impart this amount of kinetic energy to the body in five seconds. The foot-pounds of work to be done per second equal $4975 \div 5 = 995$, and the number of foot-pounds per minute equals $995 \times 60 = 59,700$. As one horsepower is equal to 33,000 foot-pounds per minute, the number of horsepower required equals $59,700 \div 33,000 = 1.8$, approximately.

Distinction between Mass and Weight. — Mass is an absolute unit; it measures the amount of matter in a body. Weight, on the contrary, is a measure of the earth's attraction (commonly called "gravity") for a body. So long as the amount of matter in a body is not changed, its mass remains unaltered; its weight, however, may change very materially, depending upon the latitude of the place where the body is weighed, the altitude (distance above or below sea level), and the temperature and barometric pressure, if weighed in air. If weighed in air under the same conditions, two bodies may weigh alike and still have different masses. For instance, a pound of iron and a pound of wood, both having been weighed in air at the same instant, have different masses, the pound of wood containing more matter than the pound of iron. The wood would weigh more than the iron in a vacuum. The reason the two weigh the same in air is that the wood has a greater volume; this causes it to displace more air than the iron, the result being that it is buoyed up more than the iron. The effect is the same, though not so marked, as if both had been placed in water. The effect of a force in changing the velocity of a moving body depends solely upon the mass; it is independent of the weight of the body. The mass of a body equals its weight divided by the acceleration due to gravity or by 32.16, which is the value at sea level, or:

$$\text{Mass} = \frac{\text{weight}}{32.16}.$$

Efficiency of Machines. — Speaking in general terms, the efficiency of a machine may be defined as the ratio of the work delivered by the machine to the work supplied to it. For instance, if 75 foot-pounds of work or energy are supplied to a machine and the machine can deliver only 60 foot-pounds of useful work, the machine is said to have an efficiency of $60 \div 75 = 0.80$, or 80 per cent. It frequently happens, however, that the work will be proportional to a force or some other quantity, in which case the efficiency may be measured by a comparison of two forces or other quantities. For instance, referring to Fig. 6, let P be a force acting on one end of a rope that passes over a pulley and has a weight Q attached to the other end. If P moves through a distance p, Q will move through a distance q, and, by the principle of virtual velocities, $Pp = Qq$, when it is assumed that there are no wasteful resistances, such as friction of the bearings, bending of the rope, etc. The efficiency in this case would evidently be 1, or 100 per cent. Since, however, there are wasteful resistances, they may be represented by W and the distance through which they act by w; consequently, the foregoing expression becomes $Pp - Ww = Qq$, or $Pp = Qq + Ww$, and the expression for efficiency becomes $\frac{Pp}{Qq + Ww}$.

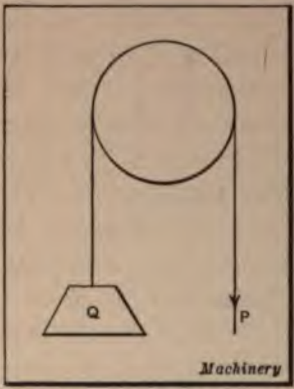


Fig. 6. Diagram for Demonstrating how the Efficiency of a Machine is determined

Since every machine or part of a machine offers wasteful resistances, the efficiency must always be less than 100 per cent, i.e., it must always be a fraction less than unity. Referring again to Fig. 6, let P_0 be the force required to move the load Q when wasteful resistances are neglected, and let P be the force actually required to move the load; then the efficiency may be defined as $e = \frac{P_0}{P}$, in which e is the efficiency. If a machine is made up of a number of separate parts, the *efficiency* of the entire machine is the product of the efficiencies of the several parts. In the case of any heat engine, the energy of the working fluid (gas, air, steam, etc.) is proportional to the temperature; hence, if T_1 is the temperature of the fluid as it enters and T_2 the temperature on leaving (both absolute), the thermal efficiency is $T_1 - T_2 \div T_1$.

Mechanical Efficiency and Effectiveness. — The theory of the efficiency of machines is one of the simplest in applied mechanics, but it nevertheless seems to be one very often misunderstood and misapplied. The hundreds of inventors of perpetual motion machines are notorious examples of those who misunderstand it and the great principle of conservation of energy, but they may be classed as impractical men of little or no importance in the machine building world. There is another large class, however, whose ideas are embodied in machines built, sold and used with various degrees of satisfaction who are more or less hazy on certain fundamental principles. It is important that this class thoroughly understand the general principles which conserve power, reduce wear, and tend generally to promote the life and efficiency of machines.

A machine may be effective without being mechanically efficient, and again it may be mechanically efficient without being effective. This is an apparent paradox generally understood and appreciated. A worm-gear, as ordinarily made, is effective but not efficient, and on the other hand if highly efficient it fails to be effective as a brake — a most important consideration in some machines. In the case of machine tools, mechanical efficiency is ordinarily regarded as a minor matter, while accuracy, convenience of operation, adaptability, safety, and pleasing lines are of paramount importance. But mechanical efficiency, aside from power saving, is important, nevertheless, as a mechanically inefficient machine wears rapidly and requires more lubrication to do its work than the efficient machine.

The efficiency of a machine is measured by the percentage of useful work available after transformation in the machine. The percentage is rarely over 95 in the simplest mechanisms, and is often less than ten.

A stiff machine in which the train of mechanism is well supported is more efficient, other things being equal, than one having a weak and flexible train. Work is lost in bending the parts, especially when the action is intermittent. A reciprocating motion may be transmitted, for example, through a lever so weak and flexible that all the work put into the machine is lost in deflecting this member, thus producing distortion and heat. Take, for example, a compressed air riveting machine of the alligator type. The stiffness of the levers is an important factor in its efficiency. A riveter having ample cylinder capacity might, nevertheless, be so weak in the levers that the toggle action would fail to produce the squeeze necessary to upset the rivets. The work that should be expended on the rivets is lost in friction of the pivots and in springing the levers which yield at the critical position so much that the necessary force to upset them is not developed. This machine would use as much compressed air as another of the same size but with stiffer mechanism which would effectively set the rivets. One is efficient and the other is totally inefficient.

Action and Reaction. — If a spring scale is held in the hands and is made to indicate 50 pounds, have 100 pounds of energy been exerted (50 pounds being resisted with one hand while pulling 50 pounds with the other), or are the pull and resistance equally divided in each hand, 25 pounds pressure being exerted in each?

According to the third law of motion, action and reaction are equal and opposite. The force indicated by the spring is the action, and if this be 50 pounds, the reaction must also be 50 pounds; hence, each hand exerts a force of 50 pounds. That this statement is correct may easily be shown by means of a simple experiment. Referring to Fig. 7, let S be a spring scale and A and B two pulleys, the centers of which are the same distance from the floor level. Then, in order that the scale may not move bodily, the weight P must be equal to the weight W; but the pull registered by the scale will be that of only one of the loads. Here P represents the left hand and W the right hand.

Measuring the Force of a Blow. — The energy with which a hammer or a falling weight strikes a body can be expressed in foot-pounds, but not directly in pounds, although the *average* force of the blow may be expressed in pounds. The number of foot-pounds of work done by a falling body equals the weight of the body multiplied by the height through which it falls. To determine the average force of the blow,

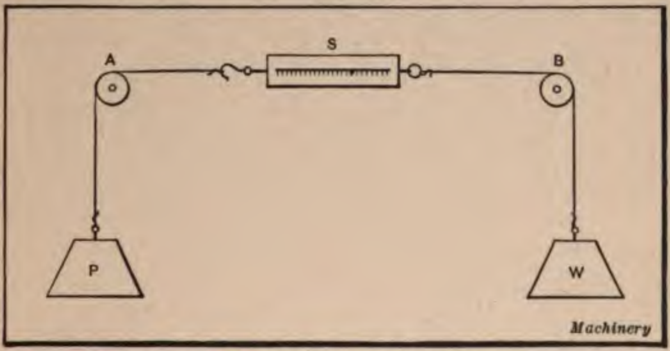


Fig. 7. Spring Scale Illustration of Third Law of Motion

it is necessary to consider the effect of the falling weight upon the body struck. For instance, if the weight fell upon a pile and drove it a certain distance into the ground, the energy of the weight in foot-pounds would be divided by the distance the pile was driven downward, or by the "amount of penetration," in obtaining the average force of the blow.

Rule: The average force of a blow is equivalent to the energy of the falling body in foot-pounds divided by the amount of penetration, plus the weight of the falling body. If

W = weight of the falling body in pounds;
H = height in feet through which it falls;
P = amount of penetration, or distance that the part struck is compressed;

then

$$\text{average force of blow} = \frac{W \times H}{P} + W.$$

Example. — The head or hammer of a pile-driver weighs 300 pounds, and it falls 10 feet upon a pile which is driven 4 inches into the ground. What is the average force of the blow?

The force of the blow in foot-pounds equals $300 \times 10 = 3000$ foot-pounds. This energy is expended over a distance of 4 inches, or $\frac{1}{3}$ foot. Hence, the average force of the blow equals $3000 \div \frac{1}{3} + 300 = 9000 + 300 = 9300$ pounds.

While the foregoing method of calculating the force of a blow is satisfactory for a pile-driver which simply has a falling weight, the force of a blow struck by a steam-hammer, or by a hammer held in the hand, depends upon additional force. If the weight of the hammer and its velocity are known, the average force of the blow may be determined, although ordinarily it would be difficult to determine the velocity of the hammer at the instant it struck the object. To illustrate this method of calculating the force of a blow, assume that the head of a steam-hammer and the parts attached to it (piston-rod and piston) weigh 1000 pounds and that a heated block of steel is reduced $\frac{1}{2}$ inch in height when the hammer strikes it with a velocity of 30 feet per second. What is the average force of the blow?

The kinetic energy of the hammer blow is first determined. Kinetic energy $= \frac{WV^2}{64.32}\cdot$ Therefore, in this particular case, the kinetic energy equals:

$$\frac{1000 \times 30^2}{64.32} = 14{,}000 \text{ foot-pounds.}$$

The block of steel was reduced $\frac{1}{2}$ inch, or $\frac{1}{24}$ foot; hence, the average force of the blow equals $14{,}000 \div \frac{1}{24} + 1000 = 337{,}000$ pounds.

The accuracy of calculating the average force of the blow by dividing the energy in foot-pounds by the penetration is affected to some extent, because the materials receiving the blow have a certain amount of elasticity and do not entirely retain the shape they have at the instant of the greatest compression.

One method of determining the force of a hammer blow is to strike a plug of lead and note the amount it is compressed, and then find how much pressure in pounds is required to compress a duplicate plug a similar amount. The lead plugs should, of course, be of the same shape and size. For instance, if a cylindrical plug 1 inch in diameter and 1 inch high is compressed a given amount by striking it one blow with a hammer, a duplicate plug is put into a testing machine and is subjected to a pressure great enough to compress it the same amount. The pressure applied to the lead plug by the testing machine is a fair indication of the force of the blow.

Force required for Bending. — Practically all text-books on mechanics treat bending stresses and resistance to bending from the viewpoint that bending is to be resisted. In the design of most machines and structures, this viewpoint is perfectly correct, but occasionally it is necessary to determine, approximately at least, the force that will actually bend a bar or a beam. Ordinary beam formulas apply just the same, whether the beam is to bend or to resist bending. In the latter case, however, the stress on the extreme or outer fibers of the beam (where the stress is greatest) must be less than the elastic limit of the material. In fact, the fiber stress should be considerably less than the elastic limit in order to avoid excessive deflection of the beam and to make it safe.

On the contrary, when a bar or plate is to be bent, the applied force must be great enough to overcome the elastic limit of the material. Hence, in the following rules and formulas for determining the force required for bending, a value is employed which is assumed to be equal to, or slightly greater than, the elastic limit of the material. The elastic limit of steel is generally assumed to be one-half the ultimate tensile strength, although it might be two-thirds of the ultimate strength in the case of nickel steel and heat-treated forgings. It is evident, then, that the force required for bending, as determined by calculations, is only approximate.

Bending a Bar supported at the Ends. — The method of calculating force required for bending depends upon the way the bar or plate to be bent is supported, and how the force is applied. Ordinarily, the parts to be bent are either supported at both ends and the force is applied in the center, or the work is held rigidly at one end and the force or pressure is applied at the other. To illustrate the first case mentioned, suppose a structural steel bar $\frac{3}{4}$ inch thick and $1\frac{1}{2}$ inch wide is supported at points 18 inches apart, as shown at *A*, Fig. 8. If the force *F* is applied at a point midway between the supports, how many pounds pressure will be required for bending the bar?

The tensile strength of structural steel is about 60,000 pounds per square inch, and if the elastic limit is assumed to be 35,000 pounds per square inch, this will doubtless be somewhat greater than the actual elastic limit. The load in pounds required for bending may be determined as follows:

Rule: Multiply the square of the thickness of the bar (vertical dimension) by twice its width, in inches, and multiply the product by the value assumed for the elastic limit. Then divide this product by three times the distance (in inches) between the supports.

Applying this rule to the example given, the square of the thickness, or 0.75, equals 0.5625, and $0.5625 \times 1.5 \times 2 = 1.6875$. The assumed value for the elastic limit, or $35{,}000 \times 1.6875 = 59{,}062$. Dividing by three times the distance between the supports, we have $59{,}062 \div 3 \times 18 = 1093$ pounds. This figure is, of course, only approximate. The actual load required for bending would probably be a little less than the calculated load, especially if the value assumed for the elastic limit is somewhat greater than the actual elastic limit.

If the foregoing rule is expressed as a formula in which:

F = force in pounds required for bending;

S = stress in pounds per square inch, which exceeds somewhat the elastic limit of the material;

B = width of the bar in inches;

T = thickness of the bar in inches, or its vertical dimension;

L = distance between supports in inches;

then

$$F = \frac{S \times 2 \times B \times T^2}{3\,L}.$$

Bending a Bar held at One End. — When the bar or plate to be bent is held at one end and the force is applied at the other, the pressure in pounds required for bending may be found as follows:

Rule: Multiply the square of the thickness of the material (vertical dimension) by its width in inches and the product by the value assumed for the elastic limit. Divide the product

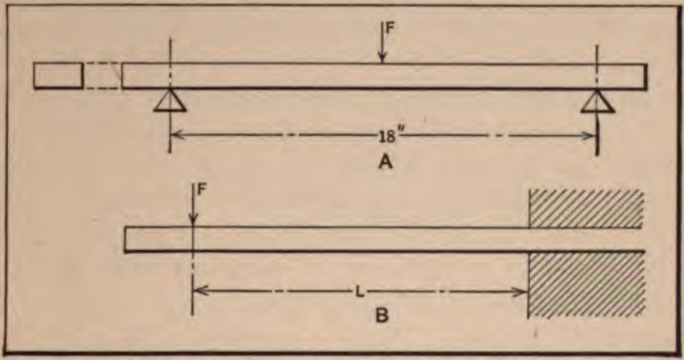


Fig. 8. (*A*) Bar supported at Each End. (*B*) Bar supported at One End

thus obtained by six times the distance (in inches) from the point where the pressure is applied to the point where the bend will occur. (Equals distance L in Fig. 8.) The result will equal the required force in pounds.

Example. — A steel plate $\frac{1}{2}$ inch thick and 6 inches wide is to be bent and the force is to be applied $13\frac{1}{2}$ inches from the clamping jaws or from the center of the bend. It is assumed that the elastic limit is 35,000 pounds per square inch and that the plate to be bent is held rigidly on each side of the place where the bend is required, as shown at B in Fig. 8.

The square of the thickness equals $0.5^2 = 0.25$. Multiplying by the width of the plate, $0.25 \times 6 = 1.5$, and $1.5 \times 35{,}000 = 52{,}500$. Dividing by 6 times the distance L, gives $52{,}500 \div 81 = 650$ pounds, nearly. If

F = force in pounds required for bending;
T = thickness of the plate, in inches;
B = width of the plate, in inches;
L = distance between point where pressure is applied and center of bend;
S = stress in pounds per square inch, which slightly exceeds elastic limit of material;

then

$$F = \frac{S \times B \times T^2}{6L}.$$

The values of S will vary considerably for different grades of steel, and also with the condition of the steel. For instance, unannealed steel will have a higher elastic limit than annealed steel; consequently, the force required for bending can be calculated only approximately.

Expansion of Metals due to Heat. — Practically all substances expand when heated and contract when cooled. The expansion of solid bodies in a longitudinal or lengthwise direction is known as *linear expansion*, and an increase in volume is known as *volumetric*, or *cubical expansion*. This expansive property of metals is taken advantage of in assembling certain parts, such as the tires of locomotive driving wheels, or other parts which must fit together tightly and which can be shrunk together more rapidly than by assembling with a hydraulic or other press. This expansive property may also prove troublesome at times, especially when a machinist or a toolmaker is finishing some part which must be very accurate. For instance, if a plug gage which has become heated by the friction of grinding is ground to the required dimension, it may shrink below the required diameter as it cools, if the expansion is not allowed for or considered. For this reason, it is often necessary to cool gages, or other accurate parts, down to an ordinary room temperature before the size is measured. There are many other classes of work in connection with which expansion and contraction should be taken into consideration.

If we know the amount that a steel rod will lengthen when its temperature is increased one degree F., the expansion for a greater increase of temperature may be determined readily. In engineering handbooks, tables will be found which give the linear expansion of different metals and other materials, per unit of length, for an increase in temperature of one degree. This figure, which is called the "coefficient of expansion," is obtained by dividing the amount that a rod of given length expands, after a one-degree rise in temperature, by the original length of the rod. For instance, if a rod 120 inches long expanded 0.0008 inch due to a one-degree F. rise in temperature, the coefficient of the linear expansion, or linear expansion per unit of length per degree F., would equal

$$\frac{8}{10{,}000} \div 120 = \frac{8}{10{,}000} \times \frac{1}{120} = \frac{8}{1{,}200{,}000} = 0.00000666.$$

Therefore, a rod made of this particular material would increase 0.00000666 of its length for each rise in temperature of one degree F. Hence, the total amount of linear expansion may be determined by the following rule:

Rule: Multiply the length of the rod or other part by the coefficient of expansion for that particular metal, and multiply the product by the difference between the original temperature and the temperature after heating.

If

L = original length of rod, or other part;
E = coefficient of linear expansion;
A = amount of expansion;
T = number of degrees F., of temperature change;

then the amount of expansion may be determined by the following formula:

$$A = L \times E \times T.$$

The coefficient of linear expansion for cast iron is given in MACHINERY'S HANDBOOK as 0.00000556, and for steel, 0.00000636. (The coefficient of volumetric expansion equals three times the linear expansion.)

Example. — If a steel end-measuring gage, 18.020 inches long, is left near a furnace, and its temperature increases from 70 to 90 degrees F., how much will the length be increased?

As the linear expansion for steel is given as 0.00000636, the gage is lengthened by the 20-degree rise of temperature an amount equal to $18.020 \times 0.00000636 \times 20 = 0.0023$ inch, approximately.

Example. — A tire is to be shrunk to a locomotive driving wheel center which is 62 inches in diameter. The tire is bored to a diameter of 61.934 inches, 0.066 inch having been allowed for the shrinkage fit. If the tire is to be expanded until it is 0.004 inch larger than the wheel center, so that it can easily be placed in position for shrinking, to what temperature must it be heated?

In solving this problem, it will be assumed that the original temperature of the tire is 70 degrees F. A total expansion, or increase in diameter, of about 0.070 is required ($62.004 - 61.934 = 0.070$), and the diameter will be considered the same as a linear dimension. The formula previously given for determining the amount of expansion may be transposed, so that the change of temperature required for a given amount of expansion can be determined. Thus, if

$$A = LET$$

then

$$T = \frac{A}{LE}.$$

Now, $A = 0.070$; $L = 61.934$; $E = 0.00000636$.

Therefore,

$$T = \frac{0.070}{61.934 \times 0.00000636} = 177 \text{ degrees.}$$

Effect of Leverage on Resistance to Shear. — A flywheel fitted with the safety device in the form of a shear pin is shown in Fig. 9. A one-inch square steel pin *C* is held between steel bushings, one bushing being held in the flywheel arm and the other in a spider *D* keyed to the shaft *B*. The flywheel *A* is free to revolve around the shaft, in case an overload should shear off pin *C*. It is assumed that 60,000 pounds would be required to shear a steel pin one inch square and also that the pin would be severed when sheared about one-third its thickness. The question is, does the distance *F* affect the resistance of the shear pin?

The position of the shear pin positively affects its effective shearing resistance. If it is located at the hub, its resistance to the action of the flywheel will be much less effective than if it is located in the rim. If the shear pin is located 20 inches from the shaft center, its effective resistance to check the flywheel will be twice that if placed only 10 inches from the center. The principle is exactly the same as found in a pair of shears. If a thick wire is to be sheared, it should be placed as near the pivot or hinge of the shears as possible in order to secure the most effective leverage.

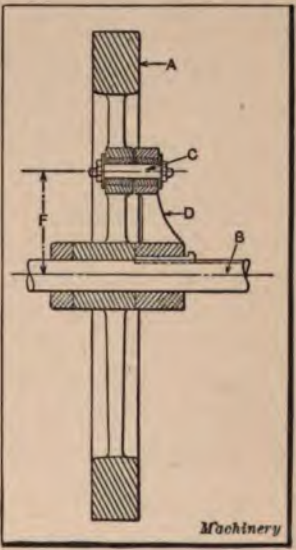


Fig. 9. Flywheel loosely mounted on Shaft and Driven by Shear Pin

Stress on Pulley Axle. — In the diagram, Fig. 10, a pulley is shown rigidly fastened to a support at a height $AF = 9$ feet from the floor. A rope passing over the pulley has one end attached to a staple *S* at a distance of 8 feet from *F*, and to the other end is attached a weight *P* of 50 pounds. Does the total stress acting on the pulley axle exceed 50 pounds?

The solution of this problem may be made clearer by first considering the view to the right. One weight *Q* of 50 pounds exactly balances the other weight *P* of 50 pounds, and since the pulley supports both weights, the total stress on the axle is $50 + 50 = 100$ pounds. Suppose, now, that one of the weights were removed and that the free end of the rope were fastened; then, in so far as the stress on the axle were concerned, the conditions would be exactly the same as before, provided the two parts of the rope were parallel. Thus, suppose the weight Q were removed; the weight P tends to fall and pull the rope along with it, but this is resisted by fastening the free end of the rope. As a result, P acts downward and the reaction acts upward; on the other side, P acts upward and the reaction acts downward. The reaction in the second case corresponds in every respect to the force Q; hence, the total stress on the axle is 100 pounds, as before.

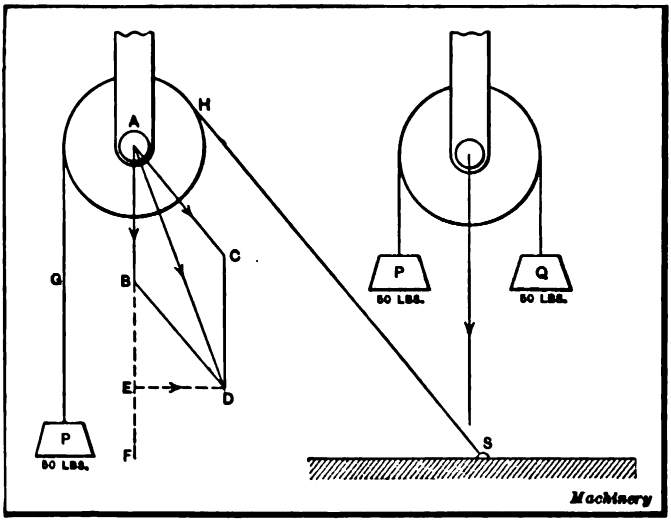


Fig. 10. Diagram illustrating Method of determining Stress on Pulley Axle

Under the conditions shown in the left-hand view, a part of the reaction of the staple tends to draw the pulley away from the perpendicular, and the stress on the axle is less than the sum of the stresses in the two parts of the rope. To find what this stress is, draw a line, as AB, parallel to the part G of the rope, and make it of a length that will represent 50 pounds; draw AC parallel to HS, the other part of the rope, and make it of the same length, also representing 50 pounds; complete the parallelogram $ACDB$, and draw diagonal AD; this measured to the same scale as AB gives the stress on the axle and shows the direction in which it acts. By prolonging AB and drawing DE perpendicular to it, ED represents the force tending to draw the pulley from the perpendicular, and AE represents the downward force on the axle. Assuming the pulley to be 1 foot in diameter, the following values were obtained in this case, by the measurement of a diagram:

$$AD = 94.25 \text{ pounds;}$$
$$AE = 88.75 \text{ pounds;}$$
$$ED = 31.75 \text{ pounds.}$$

These values agree closely with those obtained by calculation.

Horsepower transmitted by Belting. — The amount of power which a belt of given size will transmit depends upon several factors, but principally upon the speed of the belt and the amount of working stress or *effective pull* to which the belt may properly be subjected. The working stress depends upon the kind of belt, and in selecting a value for the working stress, the durability of the belt and cost of repairs should be considered. The term "effective pull," which is used in the following rule for obtaining the approximate number of horsepower that a belt will transmit, represents the difference in tension between the tight and the slack sides of the belt.

Rule: The number of horsepower that a belt of given size will transmit may be determined by multiplying the effective pull in pounds per inch of belt width by the width of the belt in inches and the belt speed in feet per minute, and then dividing the product by 33,000.

There is a difference of opinion regarding the working stress to which a belt should be selected, but the following values are commonly used: For single belts, the effective pull should be 35 pounds per inch of width and for double belts, from 55 to 65 pounds per inch of width. In the following formula:

D = diameter of driving pulley in inches;
N = number of revolutions of pulley per minute;
S = effective pull of belt per inch of width in pounds;
W = width of belt in inches.

Then,

Horsepower transmitted equals $\frac{S \times 3.14\,DNW}{12 \times 33{,}000}$, or $\frac{SDNW}{132{,}000}$ approximately.

Example. — If the effective pull on a belt is 35 pounds per inch of width, the diameter of the driving pulley, 20 inches, the number of revolutions per minute, 150, and the width of the belt, 3 inches, about what number of horsepower can be transmitted by this belt?

Horsepower transmitted equals $\frac{35 \times 20 \times 150 \times 3}{132{,}000} = 2.4$ horsepower.

In some cases, the problem is to determine the width of the belt for transmitting a given amount of power, and this may be done by simply transposing the formula previously given. Thus:

$$\text{Width of belt} = \frac{\text{horsepower to be transmitted} \times 132{,}000}{SDN}.$$

To illustrate the use of this formula, suppose three horsepower is to be transmitted and the effective pull, diameter of pulley, and its speed are the same as given in the preceding example. What width of belt is required?

$$\text{Width of belt} = \frac{3 \times 132{,}000}{35 \times 20 \times 150} = 3\tfrac{3}{4} \text{ inches, approximately.}$$

CHAPTER XVII

THE USE OF DIAGRAMS

DIAGRAMS are used for obtaining unknown factors in a problem without carrying out the calculations required in figures; they may also be used for checking the results of calculations made by figures. The results are obtained by simply following the lines in the diagram in a certain manner, which may be different for different diagrams. Each diagram covers a large number of problems of the same type, but for different kinds of problems other diagrams must be devised.

Figs. 1 to 4, inclusive, show four different kinds of diagrams, and the use of each will be taken up in detail. The four diagrams shown include the most common types, and when the student has grasped the idea of the use of these diagrams, no difficulty should be experienced in using similar diagrams for other purposes.

Diagram of Diametral Pitch, Pitch Diameter, and Number of Teeth in Spur Gears. — The diagram shown in Fig. 1 can be used for finding any one of the three quantities, number of teeth, diametral pitch, and pitch diameter of spur gears, if two of these are known.

Example. — Assume that the number of teeth in a gear is 72 and the diametral pitch 6. To find the pitch diameter from the diagram, locate the number 72 (the number of teeth in the gear) on the lower scale in the diagram and locate the diametral pitch 6 on the left-hand vertical scale. Then follow the vertical line from 72 and the horizontal line from 6, until these lines meet or intersect, and from the point of intersection follow the diagonal line to the scale on the right-hand side, where the pitch diameter, in this case 12 inches, is read off. The working of the problem is shown by the dotted lines.

If the pitch diameter is 12 inches, and the gear has 72 teeth, the diametral pitch is found by following the vertical line from 72 teeth until it intersects the diagonal line for 12-inch pitch diameter; from the point of intersection follow the horizontal line to the left-hand scale for diametral pitch, where the diametral pitch 6, is read off.

If a 6-diametral pitch gear has a pitch diameter of 12 inches, then the number of teeth in the gear can be found by following the horizontal line from 6 diametral pitch until

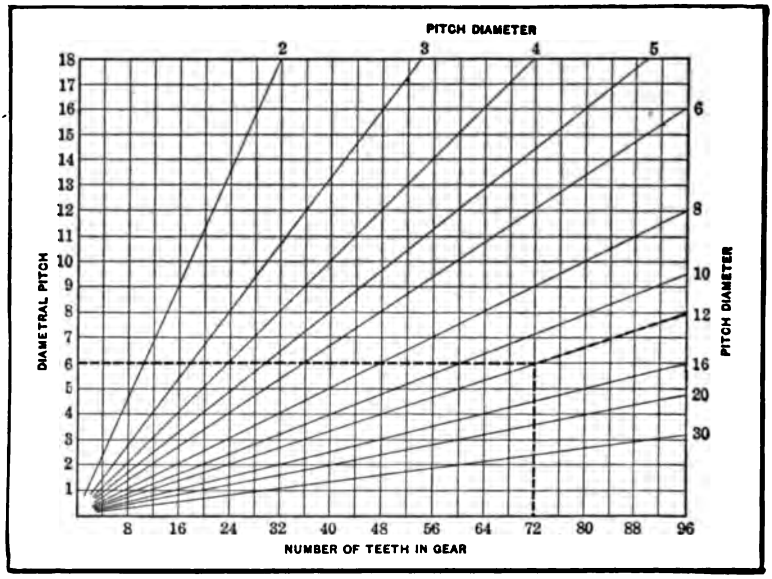


Fig. 1. Diagram of Diametral Pitch, Pitch Diameter, and Number of Teeth in Spur Gears

it intersects the diagonal line from 12 inches pitch diameter. From the point of intersection follow the vertical line down to the number of teeth, which in this case is 72.

Example. — Assume that the pitch diameter of a gear is 6 inches and the diametral pitch 8. How many teeth are there in the gear?

By following the horizontal line from 8 diametral pitch until it intersects the diagonal line for 6-inch pitch diameter, and then from the point of intersection following the vertical line to the bottom scale for the number of teeth, 48 is read off on this scale; this is the number of teeth in the gear.

If the number of teeth is not marked on the lower scale, the graduation which corresponds to the number of teeth must be estimated between the graduations marked. Thus the line for 60 teeth is the line drawn between the line for 56 and 64 teeth, and 62 teeth would be located between the line corresponding to 60 teeth and the line marked 64.

Should the vertical line from the number of teeth intersect the horizontal line from the scale for diametral pitch at a point which does not fall on one of the drawn diagonal lines, a diagonal line may be imagined as drawn between the nearest two diagonal lines shown, and the pitch diameter to which this line corresponds must be estimated. It is evident that in many cases only approximate results can be obtained, due to the fact that it is not possible to draw diagonal lines for all possible diameters, or to read small graduations on the various scales in the diagram. When very accurate results must be obtained, diagrams cannot, therefore, be relied upon entirely; but even then they are very useful for an approximate checking of the results obtained by calculations.

Diagram of Feed of End Mills. — The diagram shown in Fig. 2 may be used for obtaining the feed per minute of an end mill when the number of revolutions per minute and the feed per revolution are given.

Example. — Assume that an end mill makes 150 revolutions per minute, and that the feed per revolution is 0.040 inch. To find the feed per minute from the diagram, follow the vertical line from 150 revolutions per minute, marked on the bottom scale, until it intersects the horizontal line from the graduation 0.040 on the left-hand vertical scale. From the point of intersection, follow the curved line, either to the scale at the top or to the right-hand side where the feed per minute, in this case 6 inches, is read off.

Example. — Assume that the feed per minute is 6 inches and that the number of revolutions per minute is 150. To find the feed per revolution, follow the vertical line from 150 revolutions per minute, on the bottom scale, until it intersects the curve for 6-inch feed per minute, as found on the right-hand vertical scale. From the point of intersection follow the horizontal line to the left-hand vertical scale where the feed per revolution, in this case 0.040 inch, is read off.

On the bottom scale only each 50 revolutions are graduated, but it will be seen that there are five spaces between

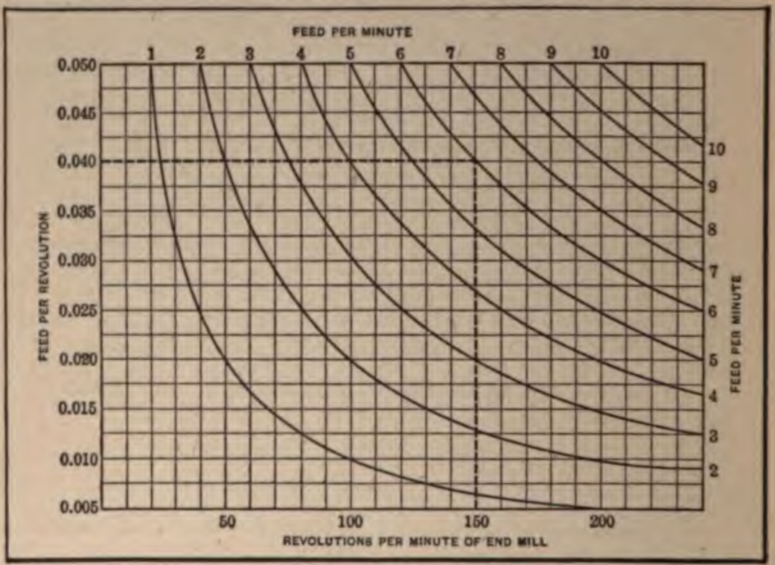


Fig. 2. Diagram showing Feed of End Mills for Different Revolutions per Minute

each number marked, and each space or graduation, therefore, is equivalent to 10 revolutions; 120 revolutions, for instance, is two spaces to the right of the 100 graduation. Find, for example, the feed per minute when the end mill makes 120 revolutions and the feed per revolution is 0.020 inch. The vertical line from 120 revolutions does not intersect the horizontal line from 0.020 inch feed at a point located on a curve drawn in the diagram, but the point of intersection falls between the two curves marked 2 and 3 inch feed per minute. As it falls almost exactly half-way between these two graduations, the feed per minute is estimated to be approximately $2\frac{1}{2}$ inches.

Diagram of Pulley Diameters. — The diagram, Fig. 3, makes it possible to find the diameter of a pulley which will run a given number of revolutions per minute, when driven by a belt from another pulley of known diameter, running at a known number of revolutions. The dotted lines in the diagram indicate the solution of an example where it is required to find the diameter of a driven pulley to run at 400

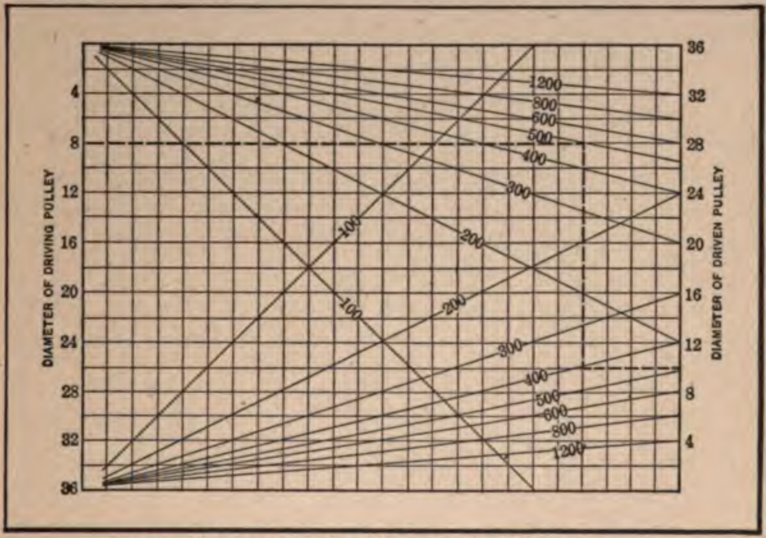


Fig. 3. Diagram of Pulley Diameters for Different Speeds

revolutions per minute, when the driving pulley is 8 inches in diameter and runs 500 revolutions per minute. In solving this problem, first find the diameter of the driving pulley on the scale to the left, and then follow the horizontal line from the point located until this line intersects the diagonal line marked 500 from the upper left-hand corner. From the point of intersection follow the vertical line until it intersects the diagonal line from the lower left-hand corner which is marked with the number of revolutions required of the driven pulley, in this case 400. From the point of intersection with this line follow the horizontal line to the scale at the right-hand side, where the diameter of the driven pulley is read off. In this case the line comes exactly between the 8- and the 12-inch marks, so that the diameter of the driven pulley is, therefore, 10 inches.

If the diameter of the driven pulley is 10 inches and the revolutions per minute of both pulleys, 400 and 500, are known, but the diameter of the driving pulley required to be found, the example is simply worked backwards; locate 10 inches on the scale to the right, follow the horizontal line until it intersects the diagonal line from the lower corner, marked 400 (number of revolutions of the driven pulley); then, from the point of intersection, follow the vertical line until it intersects the diagonal line from the upper corner, marked 500 (number of revolutions of the driving pulley); then from the point of this intersection follow the horizontal line to the left-hand scale where 8 inches is read off as the diameter of the driving pulley. The dotted lines, of course, show the working of this problem also.

If the diameter of the driving pulley is known to be 8 inches, and the diameter of the driven pulley is 10 inches, and it is known that the driving pulley makes 500 revolutions per minute, we can find from the diagram how many revolutions the driven pulley makes. Follow the horizontal line from the graduation 8 on the scale to the left until it intersects the diagonal line 500 from the upper corner, and from the point of intersection follow the vertical line until it intersects the horizontal line from 10 inches diameter on the right-hand scale. The diagonal line from the lower corner on which the vertical and horizontal lines intersect, in this case marked 400, gives the number of revolutions per minute of the driven pulley.

Horsepower Diagram. — The diagram, Fig. 4, is used for finding the horsepower which can be transmitted safely by a shaft of known diameter, making a certain number of revolutions per minute. If the horsepower to be transmitted and the number of revolutions per minute are known, the diameter of the shaft can be found.

Example. — Assume that 20 horsepower is to be transmitted by a shaft running 300 revolutions per minute. What diameter of shaft is required for transmitting this power?

First find 20 on the horsepower scale at the left-hand side of the diagram, and follow the horizontal line from 20 until it intersects the diagonal line marked 300, which is the line for the revolutions per minute. From the point of intersection follow the vertical line to the bottom scale where 2 inches

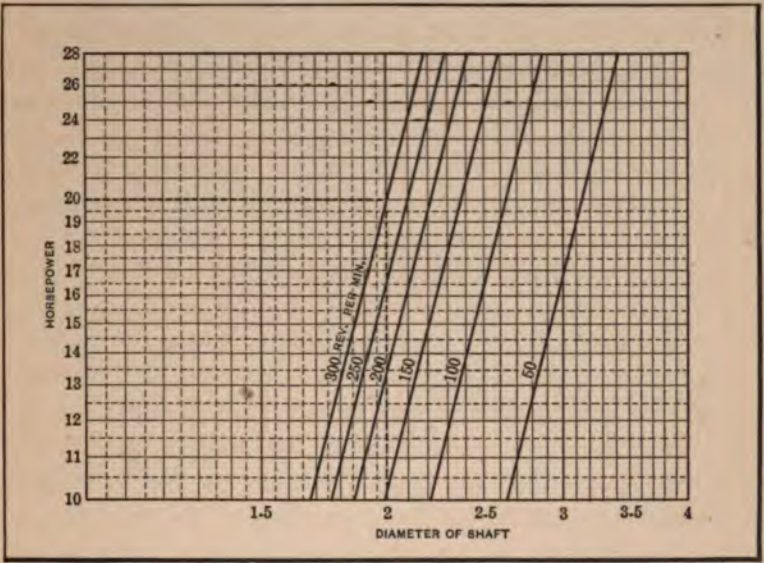


Fig. 4. Diagram showing Horsepower which can safely be Transmitted by a Shaft of Known Diameter

is read off as the required diameter of the shaft. The working of this problem is shown by dotted lines.

Example. — Assume that the diameter of a shaft is 2 inches, and that it runs at 300 revolutions per minute; then, what is the horsepower this shaft can safely transmit?

Find 2 on the scale for the diameter of shaft at the bottom of the diagram, and from this point on the scale follow the vertical line until it intersects the diagonal line for 300 revolutions per minute; then, from the point of intersection, follow the horizontal line to the left-hand scale where we read off 20, which is the horsepower that can be transmitted safely.

This diagram differs from the other three diagrams shown in that the scales are not graduated with even spacing, and it is called a logarithmic diagram, because the spacing in the scales corresponds to the logarithms of the numbers marked. This, however, has nothing to do with the reading of the diagram. It is quite as easy to use a logarithmic diagram as a diagram of any other type.

Relative Advantages of Tables and Diagrams. — There is considerable difference of opinion among mechanical men regarding the relative advantages and uses of tables and diagrams for recording data. Some engineers seem to prefer to tabulate all data that come under their observation, and that may be useful in their practice, regardless of the fact that much of this information could be more conveniently shown in diagrams. Others have what might be called the "diagram hobby," and apparently believe that all mechanical data that can possibly be given in diagram form should be put in that shape.

Both of these extreme types show a lack of appreciation of the true uses of tables and diagrams. There is, in general, a fairly well-defined field for each, and engineering data should be recorded partly by tables and partly by diagrams, according to which form best meets the requirements of the practical man. Whenever the matter dealt with has to do with parts, devices, and objects made in certain specified sizes, and has been standardized, a table is most convenient. As an example, data relating to tap drills and other dimensions required in the use of pipe taps may be mentioned. Here a diagram is of little or no value, as there are but comparatively few standard pipe tap sizes, and all the dimensions relating to each of these sizes can be tabulated easily and conveniently, with much greater exactitude than they could be put in diagram form; at the same time the chance of error in using a table is far less than in reading off a dimension from a diagram. The diagram, again, is most useful in cases where an indefinite number of combinations of values may exist, and where curves may be used to indicate the values to be found for any combination. An example of this kind is a horsepower diagram from which the horsepower that may be transmitted by gearing of different pitches and velocities may be found. In this case, tables would be entirely too voluminous, and could hardly contain all the possible combinations covered by a diagram of comparatively simple construction.

In a general way, therefore, the proper place for a table is where certain definite data are known and fixed and the values to be found corresponding to them can be put conveniently in plain figures. The diagram is preferable in all cases where a great number of different combinations of two or more initial values are given, and where a tabulation would be entirely too voluminous to be practicable, both because of the time required for compilation and the inconvenience incident to its use. The diagram has in some cases another advantage — a curve may show the trend of certain functions, indicating the rising or falling values under certain conditions, etc. In this case, the diagram is especially useful in investigating work, when making tests, or when comparing the relative efficiency of mechanisms.

INDEX